

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 46 (2000)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ORDERINGS OF MAPPING CLASS GROUPS AFTER THURSTON
Autor: SHORT, Hamish / WIEST, Bert

Kurzfassung

DOI: <https://doi.org/10.5169/seals-64801>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 24.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

ORDERINGS OF MAPPING CLASS GROUPS AFTER THURSTON

by Hamish SHORT and Bert WIEST

ABSTRACT. We are concerned with mapping class groups of hyperbolic surfaces with nonempty boundary. We present a very natural method, due to Thurston, of finding many different left orderings of such groups. The construction uses the action of the mapping class group on the boundary of the universal cover (viewed in \mathbf{H}^2), including its limit points on the circle at infinity. We classify all orderings of braid groups which arise in this way. Moreover, restricting to a certain class of “nonpathological” orderings, we prove that there are only finitely many conjugacy classes of such orderings.

We shall be concerned with certain surfaces S and their mapping class groups $\mathcal{MCG}(S)$. The surfaces under consideration are compact, with a finite set of punctures and nonempty boundary, but not necessarily oriented. We recall that $\mathcal{MCG}(S)$ is the group of isotopy classes of homeomorphisms $S \rightarrow S$ which map ∂S identically and permute the punctures. It was first proved by Dehornoy [6] that braid groups (i.e. mapping class groups of punctured disks) are left orderable. A topological proof of this result was given in [9], and the extension to mapping class groups of general surfaces with boundary can be found in [22]. (Note that mapping class groups of surfaces with empty boundary have torsion, and thus cannot be left orderable.) Here we present a very natural method, due to Thurston [24], of finding many different left orderings of such groups. In brief, one equips the surface with a hyperbolic structure, lifts it to \mathbf{H}^2 , attaches to this cover its limit points on the circle at infinity, and notices that there is a natural action of the mapping class group on the (circular) boundary of the resulting space which fixes a point, and thus an action on \mathbf{R} . We classify the set of orderings of braid groups which arise from Thurston’s construction (not all orderings do – see the example in 2.6); more precisely, we divide these orderings into two disjoint classes, which we call orderings of finite, respectively infinite, type; the orderings inside each of the classes are classified by combinatorial means. Finite type orderings are discrete, and there exist only finitely many conjugacy classes of them. By