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## A FREE GROUP ACTING ON $\mathbf{Z}^2$ WITHOUT FIXED POINTS

by SATÔ Kenzi

ABSTRACT. The group of all orientation-preserving affine transformations of the plane has a non-abelian free subgroup which stabilizes  $\mathbf{Z}^2$  and which acts on  $\mathbf{Z}^2$  without non-trivial fixed points.

### INTRODUCTION

Let  $G$  be a group acting on a non-empty set  $X$ . The following two conditions are known to be equivalent (see [D], and Theorems 4.5 and 4.8 in [W]):

- (a) *there exists a non-abelian free subgroup of  $G$  whose action on  $X$  is locally commutative;*
- (b) *there exists a  $G$ -paradoxical decomposition of  $X$  using 4 pieces, namely a partition of  $X$  in parts  $P_0, P_1, P_2, P_3$  and elements  $\alpha_0, \alpha_1, \alpha_2, \alpha_3$  in  $G$  such that*

$$X = P_0 \sqcup P_1 \sqcup P_2 \sqcup P_3 = \alpha_0(P_0) \sqcup \alpha_1(P_1) = \alpha_2(P_2) \sqcup \alpha_3(P_3).$$

Moreover, in the situation of (b), it can be shown that the subgroup of  $G$  generated by  $\alpha_0^{-1}\alpha_1$  and  $\alpha_2^{-1}\alpha_3$  is free of rank 2. (The symbol  $\sqcup$  denotes disjoint union. Recall that an action of a group  $H$  on  $X$  is *locally commutative* if the stabilizer  $\{h \in H \mid h(x) = x\}$  is commutative for all  $x \in X$ , i.e. if two elements of  $H$  which have a common fixed point commute; trivial examples of locally commutative actions are actions *without non-trivial fixed points*, for which  $\{h \in H \mid h(x) = x\}$  is reduced to  $\{1\}$  for all  $x \in X$ .)