Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 45 (1999)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: FREE GROUP ACTING ON Z^2 WITHOUT FIXED POINTS
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Kurzfassung
DOI: https://doi.org/10.5169/seals-64445

L'Enseignement Mathématique, t. 45 (1999), p. 189–194

A FREE GROUP ACTING ON $\mathbf{Z}^2$ WITHOUT FIXED POINTS

by SATÔ Kenzi

ABSTRACT. The group of all orientation-preserving affine transformations of the plane has a non-abelian free subgroup which stabilizes $\mathbf{Z}^2$ and which acts on $\mathbf{Z}^2$ without non-trivial fixed points.

INTRODUCTION

Let G be a group acting on a non-empty set X. The following two conditions are known to be equivalent (see [D], and Theorems 4.5 and 4.8 in [W]):

(a) *there exists a non-abelian free subgroup of G whose action on X is locally commutative;*

(b) *there exists a G-paradoxical decomposition of X using 4 pieces, namely a partition of X in parts P_0, P_1, P_2, P_3 and elements α_0, α_1, α_2, α_3 in G such that*

$$X = P_0 \sqcup P_1 \sqcup P_2 \sqcup P_3 = \alpha_0(P_0) \sqcup \alpha_1(P_1) = \alpha_2(P_2) \sqcup \alpha_3(P_3).$$

Moreover, in the situation of (b), it can be shown that the subgroup of G generated by ${\alpha_0}^{-1}\alpha_1$ and ${\alpha_2}^{-1}\alpha_3$ is free of rank 2. (The symbol $\sqcup$ denotes disjoint union. Recall that an action of a group H on X is *locally commutative* if the stabilizer $\{h \in H \mid h(x) = x\}$ is commutative for all $x \in X$, i.e. if two elements of H which have a common fixed point commute; trivial examples of locally commutative actions are actions *without non-trivial fixed points*, for which $\{h \in H \mid h(x) = x\}$ is reduced to $\{1\}$ for all $x \in X$.)