

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 45 (1999)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** COUNTING PATHS IN GRAPHS  
**Autor:** Bartholdi, Laurent  
**Kapitel:** 12. ACKNOWLEDGEMENTS  
**DOI:** <https://doi.org/10.5169/seals-64442>

#### Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

#### Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

#### Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

**Download PDF:** 17.05.2025

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

the amalgamated product  $\mathcal{X} = \mathcal{E} *_{\mathcal{D}} \mathcal{F}$  is isomorphic to  $\mathbf{Z}^2$ . The circuit series of  $\mathcal{D}$ ,  $\mathcal{E}$  and  $\mathcal{F}$  have been calculated explicitly and are algebraic. The circuit series of  $\mathcal{X}$  was shown in Section 10 to be transcendental; so there can exist no algebraic definition of  $G_{\mathcal{X}}$  in terms of  $G_{\mathcal{D}}$ ,  $G_{\mathcal{E}}$  and  $G_{\mathcal{F}}$ . However, there exists some relations between these series, as given by [Voi90, Theorem 5.5].

Given a graph  $\mathcal{X}$ , one can construct a graph  $\mathcal{X}^{(k)}$  on the same vertex set, and with edge set the set of paths of length  $\leq k$  in  $\mathcal{X}$ . Is there some simple relation between the path series of  $\mathcal{X}$  and of  $\mathcal{X}^{(k)}$ ? This could be useful for example to obtain asymptotics about the cogrowth of a group subject to enlargement of generating set [Cha93].

The equation (9.2) corresponds to Voiculescu's  $R$ -transform [Voi90]. His  $S$ -transform, in terms of graphs, corresponds to  $\mathcal{E} * \mathcal{F}$  with as edge set all sequences  $(e, f)$  and  $(f, e)$ , for  $e \in E(\mathcal{E})$  and  $f \in E(\mathcal{F})$ . Is there an analogue to Theorem 9.2 in this context?

Finally, (9.2) computes the circuit series of a free product in terms of the circuit series of the factors. A more complicated formula yields the path series of a free product in terms of the path series of the factors. Such considerations give another derivation of the results in Section 8.

## 12. ACKNOWLEDGEMENTS

The main result of this paper was found in Rome thanks to the nurturing of Tullio and Katiuscia Ceccherini-Silberstein and their family, whom I thank. Many people heard or read preliminary often obscure versions and provided valuable feedback; I am grateful to (in order of appearance) Michel Kervaire, Shalom Eliahou, Pierre de la Harpe, Fabrice Liardet, Rostislav Grigorchuk, Alain Valette, Étienne Ghys, Igor Lysionok, Jean-Paul Allouche, Gilles Robert, Thierry Vust, Robyn Curtis, Hung Fioramonti, and Vaughan Jones for their help and interest.

*Added in proof.* Recently Vaughan Jones has obtained very similar results in the context of planar algebras, for which some ‘path’ and ‘proper path’ series give the Hilbert-Poincaré series of a planar algebra over different subalgebras (see *Planar Algebras I*; preprint at <http://www.math.berkeley.edu/~vfr/plnalg1.ps>).