Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 45 (1999)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON GROUPS ACTING ON NONPOSITIVELY CURVED CUBICAL

COMPLEXES

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Kapitel: 5. Parallel transport in a cubical manifold and the proof of theorem 3

DOI: https://doi.org/10.5169/seals-64441

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the sequence of curves $r \circ \gamma_m$ converges locally uniformly. By Proposition 3.4, the corresponding subsequence of the sequence of unit speed geodesics γ_m converges locally uniformly. By definition, this means that the corresponding subsequence of (x_m) converges to a point $\xi \in X(\infty)$.

Let $\phi \in \Gamma$ and choose $c = c_{\phi}$ as in Lemma 4.8. Let $t_0 > 0$ be given. By Lemma 4.8 we have $r \circ \gamma_m(t_0) \in F$ for all $m \ge m_0$. By Proposition 3.4 and Lemma 4.8, we have $d(\phi(\gamma_m(t_0)), \gamma_m(t_0)) \le \sqrt{n}c_{\phi}$ for all such m. Now c_{ϕ} is independent of t_0 , hence $\phi(\xi) = \xi$.

We now complete the proof of Theorem 2 of the introduction. By Proposition 4.1, $\Delta \cong \ker h$ consists precisely of the elliptic elements of Γ . If indices of type 1 do not occur, then Proposition 4.3 applies: If k=0, then $\Gamma \cong \Delta$ fixes a point of X and possibility (1) holds. If k>0, then possibility (2) holds. If indices of type 1 occur, then possibility (3) holds by Proposition 4.9 and Corollary 4.5. Note that $\operatorname{Stab}(x) \neq \Delta$ for any $x \in X$ in this case since Δ would have a fixed point otherwise.

5. PARALLEL TRANSPORT IN A CUBICAL MANIFOLD AND THE PROOF OF THEOREM 3

Let X be a cubical manifold of dimension n. Given two chambers P and Q in X with a common face of dimension n-1, we define $t_{PQ}\colon P\to Q$ to be the *translation* which moves each point p of P along the unit geodesic segment starting at p and orthogonal to the common (n-1)-face of P to the end point in Q. The map t_{PQ} is an isomorphism and isometry of P with Q. Given a gallery $\pi=(P_1,\ldots,P_n)$ in X, the *parallel transport* along π is the isomorphism $t_\pi\colon P_1\to P_n$ given by

$$t_{\pi}:=t_{P_{n-1}P_n}\circ\cdots\circ t_{P_2P_3}\circ t_{P_1P_2}.$$

LEMMA 5.1. Let X be a simply connected cubical manifold and assume that the number of chambers adjacent to each face of codimension 2 in X is divisible by 4. Then for any two chambers P and Q in X, the parallel transport t_{π} along a gallery π connecting P and Q is independent of π .

Proof. It is enough to show that the parallel transport along any closed gallery is the identity. Let π be such a gallery with initial and final chamber P.

Represent π by a closed curve c which starts and ends in some interior point p of P, such that c misses the (n-2)-skeleton of X and crosses (n-1)-faces transversally and according to the pattern provided by π . Since X is simply connected, the curve c can be contracted in X to the point p. Since X is a manifold, the links of the vertices in X are (n-2)-connected. Hence the contraction of c can be chosen to be generic in the sense that it misses the (n-3)-skeleton of C and crosses the C-skeleton transversally. Following the curve C-along this contraction, we get a sequence of modifications of the gallery C-face of C-face of

From now on we assume that X is a simply connected cubical manifold such that the number of chambers adjacent to each face of codimension 2 in X is divisible by 4. For chambers P and Q in X define $t_{PQ} = t_{\pi}$, where π is any gallery connecting P with Q. The above lemma shows that t_{PQ} is well defined.

We fix a chamber P_0 of X and define a homomorphism $\phi \colon \Gamma \to \operatorname{Aut} P_0$ by

$$\phi(g) := t_{g(P_0)P_0} \circ g|_{P_0}.$$

The kernel $\Gamma' := \ker \phi$ is a finite index subgroup of Γ and consists precisely of those automorphisms of Γ whose restriction to any chamber commutes with the corresponding parallel transport.

COORIENTATIONS

A coorientation of a wall in a chamber is a choice of one of the two half-chambers determined by the wall. Once and for all, we choose coorientations of the walls in the above chamber P_0 . Now by Lemma 5.1, parallel transport gives rise to a consistent choice of coorientations for all walls in X.

By Corollary 1.3, X is foldable. We fix a folding and denote by Λ_i the set of hyperspaces of X with label i. Note that Λ_i is invariant under parallel transport. Along a hyperspace with label i, the half-chambers distinguished by the coorientation are all contained in the same halfspace with respect to the hyperspace. The above group Γ' preserves the families Λ_i together with the coorientations.

The index of intersection of an oriented curve c at a transversal crossing of a hyperspace $H \in \Lambda_i$ is defined to be equal to +1 or -1 respectively, according to whether the orientation of c coincides with the coorientation of d or not. Fix a point d0 in the interior of d0 which does not belong to any wall and any of the chosen coorientations. For d0 which does not belong to any wall and any of the indices of intersection of an oriented curve d0 connecting d0 and d0 with the hyperspaces from d0. Here we assume that d0 is generic, i.e. d0 does not meet the d0 does not depend on d0 since d0 is simply connected and any two such curves can be deformed into each other by a homotopy which misses the d0 skeleton of d0 and crosses the d0 skeleton of d0 and crosses the d0 skeleton of d0 transversally.

For $g \in \Gamma$ set $h_i(g) = f_i(g(p_0))$. Since the chosen system of coorientations is invariant under the action of Γ' , the maps $h_i \colon \Gamma' \to \mathbf{Z}$ are homomorphisms. We finish the proof of Theorem 3 by showing that the image of $h = (h_1, \ldots, h_n)$ is of finite index in \mathbf{Z}^n .

We need to show that the image of h contains n linearly independent vectors. To that end, we show that the image contains non-zero vectors which span arbitrarily small angles with the unit vectors e_i in \mathbb{R}^n , $1 \le i \le n$. Let σ be a unit speed geodesic ray with $\sigma(0) = p_0$ which is perpendicular in P_0 to the wall with label i. By the choice of p_0 , the ray σ does not meet the (n-2)-skeleton of X and is perpendicular to all (n-1)-faces and walls with label i which it intersects. We have

$$f_j(\sigma(m)) = \delta_{ij} \cdot m, \quad m \geq 1.$$

By the cocompactness of the action of Γ' , there is an integer $k \geq 1$ such that for any $m \geq 1$ there is a $g_m \in \Gamma'$ with $d(\sigma(m), g_m(p_0)) \leq k$. By the definition of h_j this implies $|h_j(g_m) - f_j(\sigma(m))| \leq k$. Theorem 3 follows.