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closed subschemes defined by i' and s_1, respectively. Since s_1 factors through i', we have $\mathcal{I}_{D_1} \subset \mathcal{I}_s$. Hence $D_1 - s$ is a relative effective Cartier divisor on $(X_{\mathfrak{m}} \times T_1)/T_1$ by Lemma 2.2 (b), that is, there exists a relative effective Cartier divisor $D_1{}'$ such that $D_1 = s_1 + D_1{}'$. Now we take $T_2 = D_1{}'$. We then have a finite flat morphism $T_2 \to T_1$, a section $s_2 \colon T_2 \to X_{\mathfrak{m}} \times T_2$ of the projection $X_{\mathfrak{m}} \times T_2 \to T_2$, and a relative effective Cartier divisor $D_2{}'$ on $(X_{\mathfrak{m}} \times T_2)/T_2$ such that the pull-back of $D_1{}'$ to $X_{\mathfrak{m}} \times T_2$ is equal to $s_2 + D_2{}'$. Then we take $T_3 = D_2{}', \ldots$. In this way we get finite flat morphisms $T_i \to T_{i-1}$ $(i = 1, \ldots, n)$, sections $s_i \colon T_i \to X_{\mathfrak{m}} \times T_i$, such that the pull-back of D to $X_{\mathfrak{m}} \times T_n$ is equal to $s_1 + \cdots + s_n$, where the s_i denote the relative effective Cartier divisors on $(X_{\mathfrak{m}} \times T_n)/T_n$ induced by the sections s_i. This proves our lemma.

Finally we are ready to prove Proposition 3.1.

Proof of Proposition 3.1. By Lemma A.7, there exist a finite flat morphism $\pi \colon T' \to T$ and sections $s_i \colon T' \to X_{\mathfrak{m}} \times T'$ $(i = 1, \ldots, n)$ of the projection $X_{\mathfrak{m}} \times T' \to T'$ such that the pull-back $\pi^* D$ of D to $X_{\mathfrak{m}} \times T'$ is equal to $s_1 + \cdots + s_n$. By Lemma A.6, there exists a unique morphism of schemes $f' \colon T' \to (X - S)^{(n)}$ such that the pull-back $f'^* \mathcal{D}$ of the universal relative effective Cartier divisor $\mathcal{D}$ to $X_{\mathfrak{m}} \times T'$ is $s_1 + \cdots + s_n$. Let $p_1, p_2 \colon T' \times_T T' \to T'$ be the projections. We have

$$(f'p_1)^*(\mathcal{D}) = p_1^* f'^* \mathcal{D} = p_1^*(s_1 + \cdots + s_n) = p_1^* \pi^* D = p_2^* \pi^* D = \ldots = (f'p_2)^*(\mathcal{D}).$$

that is, $(f'p_1)^*(\mathcal{D}) = (f'p_2)^*(\mathcal{D})$. By Lemma A.6 we have $f'p_1 = f'p_2$. By the theory of descent, ([SGA 1] VIII, Theorem 5.2), there exists a unique morphism of schemes $f \colon T \to (X_{\mathfrak{m}} - Q)^{(n)}$ such that $f' = f\pi$, and the pull-back of $\mathcal{D}$ to $X_{\mathfrak{m}} \times T$ is D.

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