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closed subschemes defined by  $i'$  and  $s_1$ , respectively. Since  $s_1$  factors through  $i'$ , we have  $\mathcal{I}_{D_1} \subset \mathcal{I}_s$ . Hence  $D_1 - s$  is a relative effective Cartier divisor on  $(X_m \times T_1)/T_1$  by Lemma 2.2(b), that is, there exists a relative effective Cartier divisor  $D_1'$  such that  $D_1 = s_1 + D_1'$ . Now we take  $T_2 = D_1'$ . We then have a finite flat morphism  $T_2 \rightarrow T_1$ , a section  $s_2: T_2 \rightarrow X_m \times T_2$  of the projection  $X_m \times T_2 \rightarrow T_2$ , and a relative effective Cartier divisor  $D_2'$  on  $(X_m \times T_2)/T_2$  such that the pull-back of  $D_1'$  to  $X_m \times T_2$  is equal to  $s_2 + D_2'$ . Then we take  $T_3 = D_2'$ , . . . . In this way we get finite flat morphisms  $T_i \rightarrow T_{i-1}$  ( $i = 1, \dots, n$ ), sections  $s_i: T_i \rightarrow X_m \times T_i$ , such that the pull-back of  $D$  to  $X_m \times T_n$  is equal to  $s_1 + \dots + s_n$ , where the  $s_i$  denote the relative effective Cartier divisors on  $(X_m \times T_n)/T_n$  induced by the sections  $s_i$ . This proves our lemma.

Finally we are ready to prove Proposition 3.1.

*Proof of Proposition 3.1.* By Lemma A.7, there exist a finite flat morphism  $\pi: T' \rightarrow T$  and sections  $s_i: T' \rightarrow X_m \times T'$  ( $i = 1, \dots, n$ ) of the projection  $X_m \times T' \rightarrow T'$  such that the pull-back  $\pi^*D$  of  $D$  to  $X_m \times T'$  is equal to  $s_1 + \dots + s_n$ . By Lemma A.6, there exists a unique morphism of schemes  $f': T' \rightarrow (X - S)^{(n)}$  such that the pull-back  $f'^*\mathcal{D}$  of the universal relative effective Cartier divisor  $\mathcal{D}$  to  $X_m \times T'$  is  $s_1 + \dots + s_n$ . Let  $p_1, p_2: T' \times_T T' \rightarrow T'$  be the projections. We have

$$(f'p_1)^*(\mathcal{D}) = p_1^*f'^*\mathcal{D} = p_1^*(s_1 + \dots + s_n) = p_1^*\pi^*D = p_2^*\pi^*D = \dots = (f'p_2)^*(\mathcal{D}).$$

that is,  $(f'p_1)^*(\mathcal{D}) = (f'p_2)^*(\mathcal{D})$ . By Lemma A.6 we have  $f'p_1 = f'p_2$ . By the theory of descent, ([SGA 1] VIII, Theorem 5.2), there exists a unique morphism of schemes  $f: T \rightarrow (X_m - Q)^{(n)}$  such that  $f' = f\pi$ , and the pull-back of  $\mathcal{D}$  to  $X_m \times T$  is  $D$ .

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