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$$\int_{B_0} f(\cdot, y) d\bar{\nu}_0(y) = \int_{B_0} \tilde{f}(\cdot, y) d\bar{\nu}_0(y) \quad [\bar{\mu}_0].$$

Trivially, this remains true if $[\bar{\mu}_0]$ is replaced by $[\mu]$, and an integration yields

$$(18) \quad \int_A \int_{B_0} f(x, y) d\bar{\nu}_0(y) d\mu(x) = \int_A \int_{B_0} \tilde{f}(x, y) d\bar{\nu}_0(y) d\mu(x)$$

($A \in \mathcal{A}$, $B_0 \in \bar{\mathcal{B}}_0$). We now want to interchange the order of integrations. Since \tilde{f} is trivially $\mathcal{A} \otimes \bar{\mathcal{B}}_0$ -measurable, we may obviously do this on the right hand side of (18). To do the same on the left hand side, we rewrite it successively as

$$\int_A \int_{B_0} f(x, y) d\nu(y) d\mu(x) = \int_{B_0} \int_A f(x, y) d\mu(x) d\nu(y) = \int_{B_0} \int_A f(x, y) d\mu(x) d\bar{\nu}_0(y),$$

where the last equality follows from a second application of Claim 1, with the role of the variables interchanged. Thus (18) yields

$$(19) \quad \int_{B_0} \int_A f(x, y) d\mu(x) d\bar{\nu}_0(y) = \int_{B_0} \int_A \tilde{f}(x, y) d\mu(x) d\bar{\nu}_0(y)$$

($A \in \mathcal{A}$, $B_0 \in \bar{\mathcal{B}}_0$). Now the argument leading from (17) to (18) can be repeated to lead from (19) to a corresponding statement with B in place of B_0 , ν in place of $\bar{\nu}_0$, and \mathcal{B} in place of $\bar{\mathcal{B}}_0$, which is equivalent to

$$\int_{A \times B} f d\mu \otimes \nu = \int_{A \times B} \tilde{f} d\mu \otimes \nu \quad (A \in \mathcal{A}, B \in \mathcal{B}).$$

This shows that $f = \tilde{f}$ $[\mu \otimes \nu]$, which yields the desired conclusion. \square

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