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 $Sp(1,n)/Sp(1) \times Sp(n)$   
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The techniques employed to prove the inversion formula (that is, Koornwinder's analytic continuation argument and the change of contour of integration) are the same used in [Shi] for the case of Hermitian symmetric pairs. Our choice of this method of proof is motivated by the propaedeutic nature of this paper. In fact, the computations involved in the proofs presented above are very much in the spirit of those required for the decomposition of the canonical representations in [DP].

We just mention a few alternative methods. First of all, because of Formula (7.74) and Part 3 of Proposition 4.3, the spectral theorem for the  $\tau_l$ -spherical transform can be deduced from the spectral theorem for the differential operator  $L_l$  (see (7.56)) on a suitable domain in  $L^2(\Delta(t) dt)$  on which it is self-adjoint. The latter theorem can be classically determined as an application of the Weyl-Titchmarsh Theorem. A second method is obtained observing the relation, ensured by Formula (4.39), between the  $\tau_l$ -spherical transform and the Jacobi transform. Theorems 2.3 and 2.4 of [K2] are then quickly translated to our situation. Finally, observe that Koornwinder's method with the Abel transform can also be applied directly here because of Formula (6.54).

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