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## 1. A THEOREM OF GROTHENDIECK

The following theorem is a special case of Grothendieck's theorems, and the proof can be found in [Mu] §5, [H] §3.12, or [EGA] III, §7.7.5, 7.9.4.

**THEOREM 1.1.** *Let  $q: V \rightarrow T$  be a proper flat morphism of noetherian schemes and let  $\mathcal{L}$  be an invertible sheaf on  $V$ . For each  $t \in T$  denote the fiber  $V \otimes_T \text{spec}(k(t))$  of  $q$  at  $t$  by  $V_t$ , where  $k(t)$  is the residue field of  $T$  at  $t$ . Denote the inverse image of  $\mathcal{L}$  on  $V_t$  by  $\mathcal{L}_t$ .*

- (a) *The function  $t \mapsto \chi(\mathcal{L}_t) = \sum_i (-1)^i \dim_{k(t)} H^i(V_t, \mathcal{L}_t)$  is locally constant on  $T$ .*
- (b) *For each  $i$ , the function  $t \mapsto \dim_{k(t)} H^i(V_t, \mathcal{L}_t)$  on  $T$  is upper semicontinuous.*
- (c) *If  $T$  is reduced and connected and if  $t \mapsto \dim_{k(t)} H^i(V_t, \mathcal{L}_t)$  is a constant function on  $T$ , then  $R^i q_* \mathcal{L}$  is a locally free sheaf on  $T$  and the map  $R^i q_* \mathcal{L} \otimes_{\mathcal{O}_T} k(t) \rightarrow H^i(V_t, \mathcal{L}_t)$  is an isomorphism.*
- (d) *If  $H^1(V_t, \mathcal{L}_t) = 0$  for all  $t \in T$ , then  $R^1 q_* \mathcal{L} = 0$  and  $q_* \mathcal{L}$  is a locally free sheaf. Moreover the formation of  $q_* \mathcal{L}$  commutes with any base change.*

## 2. RELATIVE EFFECTIVE CARTIER DIVISORS

Let  $q: X \rightarrow T$  be a morphism of noetherian schemes. A *relative effective Cartier divisor* on  $X/T$  is an effective Cartier divisor on  $X$  that is flat over  $T$  when regarded as a closed subscheme of  $X$ . When  $T = \text{spec}(R)$  is affine, a closed subscheme  $D$  of  $X$  is a relative effective Cartier divisor if and only if there exists an open affine covering  $U_i = \text{spec}(R_i)$  of  $X$  and  $g_i \in R_i$  such that

- (a)  $D \cap U_i = \text{spec}(R_i/(g_i))$ ;
- (b)  $g_i$  is not a zero divisor;
- (c)  $R_i/(g_i)$  is flat over  $R$ .

**REMARK 2.1.** Let  $D$  be an effective Cartier divisor on  $X/T$ , let  $\mathcal{I}(D)$  be the sheaf of ideals defining  $D$ , and let  $\mathcal{L}(D)$  be the invertible sheaf corresponding to  $D$ . We have  $\mathcal{L}(D) = \mathcal{I}(D)^{-1}$ . The inclusion  $\mathcal{I}(D) \subset \mathcal{O}_X$  induces  $\mathcal{O}_X \subset \mathcal{I}(D)^{-1} = \mathcal{L}(D)$ , hence a section  $s_D$  of  $\mathcal{L}(D)$ .