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1. A THEOREM OF GROTHENDIECK

The following theorem is a special case of Grothendieck's theorems, and the proof can be found in [Mu] §5, [H] §3.12, or [EGA] III, §7.7.5, 7.9.4.

THEOREM 1.1. *Let $q\colon V \to T$ be a proper flat morphism of noetherian schemes and let $\mathcal{L}$ be an invertible sheaf on V. For each $t \in T$ denote the fiber $V \otimes_T \operatorname{spec}(k(t))$ of q at t by V_t, where $k(t)$ is the residue field of T at t. Denote the inverse image of $\mathcal{L}$ on V_t by $\mathcal{L}_t$.*

(a) *The function $t \mapsto \chi(\mathcal{L}_t) = \sum_i (-1)^i \dim_{k(t)} H^i(V_t, \mathcal{L}_t)$ is locally constant on T.*
(b) *For each i, the function $t \mapsto \dim_{k(t)} H^i(V_t, \mathcal{L}_t)$ on T is upper semicontinuous.*
(c) *If T is reduced and connected and if $t \mapsto \dim_{k(t)} H^i(V_t, \mathcal{L}_t)$ is a constant function on T, then $R^i q_* \mathcal{L}$ is a locally free sheaf on T and the map $R^i q_* \mathcal{L} \otimes_{\mathcal{O}_T} k(t) \to H^i(V_t, \mathcal{L}_t)$ is an isomorphism.*
(d) *If $H^1(V_t, \mathcal{L}_t) = 0$ for all $t \in T$, then $R^1 q_* \mathcal{L} = 0$ and $q_* \mathcal{L}$ is a locally free sheaf. Moreover the formation of $q_* \mathcal{L}$ commutes with any base change.*

2. RELATIVE EFFECTIVE CARTIER DIVISORS

Let $q\colon X \to T$ be a morphism of noetherian schemes. A *relative effective Cartier divisor* on X/T is an effective Cartier divisor on X that is flat over T when regarded as a closed subscheme of X. When $T = \operatorname{spec}(R)$ is affine, a closed subscheme D of X is a relative effective Cartier divisor if and only if there exists an open affine covering $U_i = \operatorname{spec}(R_i)$ of X and $g_i \in R_i$ such that

(a) $D \cap U_i = \operatorname{spec}\big(R_i/(g_i)\big)$;
(b) g_i is not a zero divisor;
(c) $R_i/(g_i)$ is flat over R.

REMARK 2.1. Let D be an effective Cartier divisor on X/T, let $\mathcal{I}(D)$ be the sheaf of ideals defining D, and let $\mathcal{L}(D)$ be the invertible sheaf corresponding to D. We have $\mathcal{L}(D) = \mathcal{I}(D)^{-1}$. The inclusion $\mathcal{I}(D) \subset \mathcal{O}_X$ induces $\mathcal{O}_X \subset \mathcal{I}(D)^{-1} = \mathcal{L}(D)$, hence a section s_D of $\mathcal{L}(D)$.