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THREE DISTANCE THEOREMS AND COMBINATORICS ON WORDS

by Pascal ALESSANDRI and Valérie BERTHÉ

ABSTRACT. The aim of this paper is to investigate the connection between some generalizations of the three distance theorem and combinatorics on words for sequences defined as codings of irrational rotations on the unit circle. We also give some new results concerning the frequencies of factors for such sequences.

1. INTRODUCTION

For a given α in $]0,1[$, let us place the points $\{0\}, \{\alpha\}, \{2\alpha\}, \ldots, \{n\alpha\}$ on the *unit circle* (we mean here the circle of *perimeter* 1), where $\{x\}$ denotes as usual, the fractional part of x (i.e., if $\lfloor x \rfloor$ denotes the largest integer not exceeding x, $\{x\} = x - \lfloor x \rfloor$). These points partition the unit circle into $n+1$ intervals having at most three lengths, one being the sum of the other two. This property is known as the *three distance theorem* and can be seen as a geometric interpretation of good approximation properties of the Farey partial convergents in the continued fraction expansion of α.

The connection between this classical theorem in diophantine approximation and combinatorics on words is particularly apparent in the following result, known as the *three gap theorem*, which is equivalent to the three distance theorem and can be seen as its "dual": assume we are given α and β in the interval $]0,1[$, the gaps between the successive n for which $\{\alpha n\} < \beta$ take at most three values, one being the sum of the other two. It is indeed natural to introduce the binary sequence with values 0 and 1, defined as the coding of the orbit of a point of the unit circle under the rotation by angle α with