

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 44 (1998)  
**Heft:** 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** HOMFLY POLYNOMIAL VIA AN INVARIANT OF COLORED PLANE GRAPHS  
**Autor:** Murakami, Hitoshi / Yamada, Shuji  
**Kurzfassung**  
**DOI:** <https://doi.org/10.5169/seals-63908>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 02.03.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## HOMFLY POLYNOMIAL VIA AN INVARIANT OF COLORED PLANE GRAPHS

by Hitoshi MURAKAMI, Tomotada OHTSUKI and Shuji YAMADA

ABSTRACT. After the first discovery of the quantum invariant associated with  $SU(2)$  by V.F.R. Jones [3], the invariants associated with  $SU(n)$  were found by several authors [1]. It was first proved by V.G. Turaev [16] that all these come from so-called “quantum groups”, especially from their  $R$ -matrices corresponding to the vector representations. There also exist various quantum invariants corresponding to other representations (see for example [7], [14], [11]).

The aim of this paper is to give a graphical way to define  $SU(n)$  quantum invariants for links. To do this we first construct an invariant of colored, oriented, trivalent, plane graphs for each  $n$  ( $\geq 2$ ). Then we show that the  $SU(n)$  polynomial invariant corresponding to the vector representation (HOMFLY polynomial) can be defined by using our graph invariant.

We can also show that our invariant defines the  $SU(n)$  polynomial invariant corresponding to the anti-symmetric tensors of the vector representation.

We note that our graph invariant for  $SU(3)$  was first introduced by G. Kuperberg in [8]. The second and the third authors used it in [12] to construct magic elements and defined the quantum  $SU(3)$  invariants for 3-manifolds. Now [12] and the present paper together give an elementary and self-contained proof of the existence of magic elements for  $SU(3)$  and so that of the quantum  $SU(3)$  invariants of 3-manifolds just like W.B.R. Lickorish did for  $SU(2)$  in [9] using the Kauffman bracket [5]. See [17] for a similar approach to  $SU(n)$  invariants of 3-manifolds.

We also note that our graph invariant may be obtained (not checked yet) by direct computations of the universal  $R$ -matrix. But the advantage of our definition is that it does not require any knowledge of quantum groups nor representation theory. On the contrary we can recover the  $R$ -matrix of the quantum group  $U_q(\mathfrak{sl}(n, \mathbb{C}))$  corresponding at least to the vector representation.

---

This work was inspired after conversation with M. Kosuda and J. Murakami, to whom the authors express their gratitude.

The first named author was in the Department of Mathematics, Osaka City University when the work was carried out and partially supported by Grant-in-Aid for Scientific Research on Priority Areas 231 “Infinite Analysis”, the Ministry of Education, Science, Sports and Culture. The second named author was in the Department of Mathematical Sciences, University of Tokyo.