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## A DYNAMICAL SYSTEMS APPROACH TO BIRKHOFF'S THEOREM

by Karl Friedrich SIBURG \*)

ABSTRACT. We present a new proof of Birkhoff's classical theorem that an embedded homotopically nontrivial circle, which is invariant under a monotone twist map on  $S^1 \times \mathbf{R}$ , must be the graph of a Lipschitz function.

### 1. INTRODUCTION

Consider the two-dimensional cylinder  $S^1 \times \mathbf{R} \cong \mathbf{R}/\mathbf{Z} \times \mathbf{R}$ , respectively its universal cover  $\mathbf{R}^2$  with coordinates  $x, y$ . A diffeomorphism  $\phi: S^1 \times \mathbf{R} \rightarrow S^1 \times \mathbf{R}$  is called a monotone twist mapping if it is area-preserving and satisfies the monotone twist condition  $\partial(\pi_x \circ \phi)/\partial y \neq 0$ , where  $\pi_x$  denotes the projection onto the first coordinate. This means, in particular, that (pre-)images of verticals under any lift of  $\phi$  are graphs over the  $x$ -axis.

The twist condition is not as artificial as it might seem. Monotone twist mappings appear in a variety of situations, often unexpected and only discovered by clever coordinate choices. In the following, we give a few examples. The reader may consult [LCa, MF, Mo1, Mo2] for more detailed information and further references.

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