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A NEW PROOF OF VINCENT'S THEOREM

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ABSTRACT. Vincent's theorem (1836) asserts that, given a real polynomial $f(x)$ without multiple roots, the substitution

$$x \leftarrow c_0 + \cfrac{1}{c_1 + \cfrac{1}{c_2 + \ddots + \cfrac{1}{c_h + \cfrac{1}{x}}}}$$

where the c_i are arbitrary positive integers and h is sufficiently large, transforms $f(x)$ into a polynomial $f_{h+1}(x)$ which has at most one sign variation in the sequence of its coefficients.

This theorem is basic for highly efficient methods (implemented in modern computer algebra systems) to separate the roots of a real polynomial.

In this paper we provide a new simple proof of the theorem, which improves the known estimates of the size of h and can be extended to the case of multiple roots. We also give an historical survey of the subject.

1. INTRODUCTION

The aim of this paper is to give a new and simple proof of Vincent's theorem. The theorem has an interesting history.

It originally appeared as a note, *Sur la résolution des équations numériques*, appended at the end of the sixth edition of Bourdon's *Élémens d'algèbre* [13], without explicit mention of Vincent's authorship. Bourdon, who was Vincent's father-in-law [1]), merely acknowledges his debt to his son-in-law for "plusieurs améliorations de détail et quelques additions" in the *Avertissement* at the beginning of his book.

[1]) Information about Vincent, who was an influential personality in his time, can be found in [21] and [31].