

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 43 (1997)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: THE NORMALISER ACTION AND STRONGLY MODULAR LATTICES
Autor: Nebe, Gabriele
Bibliographie

DOI: <https://doi.org/10.5169/seals-63272>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

Download PDF: 18.05.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

the Sylow-2-subgroup of $L^\# / L$) may be derived from the equality

$$\left[(Sp_4(3) \circ C_3) \underset{\sqrt{-3}}{\boxtimes} SL_2(3) \right]_{16} = \left[(Sp_4(3) \circ C_3) \underset{\sqrt{-3}}{\boxtimes} SL_2(3) \right]_{16}^{2(2)}$$

and

$$\left[SL_2(5) \underset{\infty, 3}{\boxtimes} (SL_2(3) \square C_3) \right]_{16} = \left[(SL_2(5).2 \circ C_3) \underset{\sqrt{-3}}{\boxtimes} SL_2(3) \right]_{16}^{2(2)}$$

using Proposition 5.

Similarly one uses Proposition 5 to show the 2-modularity of the lattices of the r.i.m.f. group 6 in $GL_{24}(\mathbf{Q})$ using the description

$$\left[6.U_4(3).2 \underset{\sqrt{-3}}{\boxtimes} SL_2(3) \right]_{24} = \left[6.U_4(3).2 \underset{\sqrt{-3}}{\circ} SL_2(3) \right]_{24}^{2(2)}.$$

For the groups 44 and 64, which are the only groups which are not p -lattice sparse for a relevant prime p ($=2$), one has to note that the invariant sublattice of index 2^{12} in L is unique.

The theorem now follows from the next lemma. \square

LEMMA 9. *The lattices (of determinant $3^8 \cdot 5^8$) of the r.i.m.f. subgroup $G := [\pm \text{Alt}_6.2^2]_{16} \leq GL_{16}(\mathbf{Q})$ (number 20 of [NeP 95]) are not (strongly) modular.*

Proof. Let L be such a G -invariant lattice and $L' \in \pi(L)$. Assume that there is a similarity $s : L' \rightarrow L$. By Proposition 3, this similarity s normalises G . Let $U \cong \text{Alt}_6$ be the characteristic subgroup $\cong \text{Alt}_6$ of G . Since the full automorphism group of U is already induced by conjugation with elements of G , there exists $g \in G$, such that $n := gs \in GL_{16}(\mathbf{Q})$ centralises U . Hence $n \in C_{M_{16}(\mathbf{Q})}(U) \cong \mathbf{Q}[\sqrt{5}]$. Since this number field does not contain an element of norm 3, one concludes that $[L' : L] = 5^8$. So the lattice L is neither similar to $L^\#$ nor to the lattice $L' \in \pi(L)$ corresponding to the 3-Sylow subgroup of $L^\# / L$. Note that if $[L' : L] = 5^8$, an element $x \in C_{M_{16}(\mathbf{Q})}(U)$ with $x^2 = 5$, induces a similarity by Proposition 4. \square

REFERENCES

- [CCNPW 85] CONWAY, J. H., R. T. CURTIS, S. P. NORTON, R. A. PARKER and R. A. WILSON. *Atlas of Finite Groups*. Oxford University Press, 1985.
- [Neb 95] NEBE, G. Endliche rationale Matrixgruppen vom Grad 24. Dissertation RWTH Aachen. *Aachener Beiträge zur Mathematik* 12 (1995).

- [Neb 96] NEBE, G. Finite subgroups of $GL_{24}(\mathbf{Q})$. *Exp. Math.* 5 (1996), 163–195.
- [Neb 96a] —— Finite subgroups of $GL_n(\mathbf{Q})$ for $25 \leq n \leq 31$. *Comm. Alg.* 24 (7) (1996), 2341–2397.
- [NeP 95] NEBE, G. and W. PLESKEN. Finite rational matrix groups of degree 16. *AMS Memoirs*, vol. 116, No. 556 (1995).
- [PlN 95] PLESKEN, W. and G. NEBE. Finite rational matrix groups. *AMS Memoirs*, vol. 116, No. 556 (1995).
- [Que 95] QUEBBEMANN, H.-G. Modular lattices in Euclidean spaces. *J. Number Theory* 54 (1995), 190–202.
- [Que 96] —— Atkin-Lehner eigenforms and strongly modular lattices. *L'Ens. Math.* 43 (1997), 55–65.

(*Reçu le 25 juin 1996; version révisée reçue le 22 novembre 1996*)

Gabriele Nebe

Lehrstuhl B für Mathematik
RWTH Aachen
Templergraben 64
52062 Aachen
Germany
E-mail: gabi@willi.math.rwth-aachen.de