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the Sylow-2-subgroup of $L^\# / L$) may be derived from the equality

$$[(Sp_4(3) \circ C_3) \overset{2}{\boxtimes}_{\sqrt{-3}} SL_2(3)]_{16} = [(Sp_4(3) \circ C_3) \overset{2(2)}{\boxtimes}_{\sqrt{-3}} SL_2(3)]_{16}$$

and

$$[SL_2(5) \overset{2(3)}{\boxtimes}_{\infty,3} (SL_2(3) \overset{2}{\square} C_3)]_{16} = [(SL_2(5).2 \circ C_3) \overset{2(2)}{\boxtimes}_{\sqrt{-3}} SL_2(3)]_{16}$$

using Proposition 5.

Similarly one uses Proposition 5 to show the 2-modularity of the lattices of the r.i.m.f. group 6 in $GL_{24}(\mathbf{Q})$ using the description

$$[6.U_4(3).2 \overset{2}{\boxtimes}_{\sqrt{-3}} SL_2(3)]_{24} = [6.U_4(3).2 \overset{2(2)}{\circ} SL_2(3)]_{24}.$$

For the groups 44 and 64, which are the only groups which are not p -lattice sparse for a relevant prime p ($=2$), one has to note that the invariant sublattice of index 2^{12} in L is unique.

The theorem now follows from the next lemma. \square

LEMMA 9. *The lattices (of determinant $3^8 \cdot 5^8$) of the r.i.m.f. subgroup $G := [\pm \text{Alt}_6 . 2^2]_{16} \leq GL_{16}(\mathbf{Q})$ (number 20 of [NeP 95]) are not (strongly) modular.*

Proof. Let L be such a G -invariant lattice and $L' \in \pi(L)$. Assume that there is a similarity $s : L' \rightarrow L$. By Proposition 3, this similarity s normalises G . Let $U \cong \text{Alt}_6$ be the characteristic subgroup $\cong \text{Alt}_6$ of G . Since the full automorphism group of U is already induced by conjugation with elements of G , there exists $g \in G$, such that $n := gs \in GL_{16}(\mathbf{Q})$ centralises U . Hence $n \in C_{M_{16}(\mathbf{Q})}(U) \cong \mathbf{Q}[\sqrt{5}]$. Since this number field does not contain an element of norm 3, one concludes that $[L' : L] = 5^8$. So the lattice L is neither similar to $L^\#$ nor to the lattice $L' \in \pi(L)$ corresponding to the 3-Sylow subgroup of $L^\# / L$. Note that if $[L' : L] = 5^8$, an element $x \in C_{M_{16}(\mathbf{Q})}(U)$ with $x^2 = 5$, induces a similarity by Proposition 4. \square

REFERENCES

- [CCNPW 85] CONWAY, J. H., R. T. CURTIS, S. P. NORTON, R. A. PARKER and R. A. WILSON. *Atlas of Finite Groups*. Oxford University Press, 1985.
- [Neb 95] NEBE, G. Endliche rationale Matrixgruppen vom Grad 24. Dissertation RWTH Aachen. *Aachener Beiträge zur Mathematik 12* (1995).

- [Neb 96] NEBE, G. Finite subgroups of $GL_{24}(\mathbf{Q})$. *Exp. Math.* 5 (1996), 163–195.
- [Neb 96a] — Finite subgroups of $GL_n(\mathbf{Q})$ for $25 \leq n \leq 31$. *Comm. Alg.* 24 (7) (1996), 2341–2397.
- [NeP 95] NEBE, G. and W. PLESKEN. Finite rational matrix groups of degree 16. *AMS Memoirs*, vol. 116, No. 556 (1995).
- [PIN 95] PLESKEN, W. and G. NEBE. Finite rational matrix groups. *AMS Memoirs*, vol. 116, No. 556 (1995).
- [Que 95] QUEBBEMANN, H.-G. Modular lattices in Euclidean spaces. *J. Number Theory* 54 (1995), 190–202.
- [Que 96] — Atkin-Lehner eigenforms and strongly modular lattices. *L'Ens. Math.* 43 (1997), 55–65.

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