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(17) $$\widehat{c}_t(\bar{S})\widehat{c}_t(\bar{Q}) = 1 + a\big(\widetilde{c}_t(\bar{\mathcal{E}})\big) = 1 - p_a(t)\,,$$

where the subscript t denotes the corresponding Chern polynomial. Multiplying both sides of (17) by $1 + p_a(t)$ and using the properties of multiplication in $\mathcal{A}$ gives the equivalent form

(18) $$\widehat{c}_t(\bar{S}) * \widehat{c}_t(\bar{Q}) * \big(1 + p_a(t)\big) = 1\,.$$

We now note that the *harmonic number generating function*

$$\sum_{i=0}^{\infty} \mathcal{H}_i t^i = \frac{t}{1-t} + \frac{1}{2}\frac{t^2}{1-t} + \frac{1}{3}\frac{t^3}{1-t} + \cdots = \frac{\log(1-t)}{t-1}\,.$$

It follows that

$$p_a(-t) = \sum_{i=0}^{\infty} \mathcal{H}_i p_i(y) t^{i+1} = t \sum_{j=1}^{s} \sum_{i=0}^{\infty} \mathcal{H}_i (y_j t)^i = -t \sum_{j=1}^{s} \frac{\log(1-y_j t)}{1-y_j t}$$

and thus

$$p_a(t) = t \sum_{j=1}^{s} \frac{\log(1+y_j t)}{1+y_j t}\,.$$

Substituting this in equation (18) gives relation $\mathcal{R}_2$. □

Theorem 6 shows that the relations in the Arakelov Chow ring of G are the classical geometric ones perturbed by a new "arithmetic factor" of $1 + p_a(t)$. While this factor is closely related to the power sums $p_i(\bar{Q})$, the most natural basis of symmetric functions for doing calculations in $CH(G)$ is the basis of Schur polynomials (corresponding to the Schubert classes; see for example [F], § 14.7). The arithmetic analogues of the special Schubert classes involve the power sum perturbation above; multiplication formulas are thus quite complicated (see [Ma]).

In geometry the Chern roots x_i and y_j all "live" on the complete flag variety above G. There are certainly natural line bundles on the flag variety whose first Chern classes correspond to the roots in Theorem 6. However on flag varieties the situation is more complicated and our knowledge is not as complete. We refer the reader to [T] for more details.

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