Zeitschrift: L'Enseignement Mathématique

Herausgeber: Commission Internationale de l'Enseignement Mathématique

Band: 43 (1997)

Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: BOTT-CHERN FORMS AND ARITHMETIC INTERSECTIONS

Autor: Tamvakis, Harry

Bibliographie

DOI: https://doi.org/10.5169/seals-63270

(17) $$\widehat{c}_t(\bar{S})\widehat{c}_t(\bar{Q}) = 1 + a\big(\widetilde{c}_t(\bar{\mathcal{E}})\big) = 1 - p_a(t)\,,$$

where the subscript t denotes the corresponding Chern polynomial. Multiplying both sides of (17) by $1+p_a(t)$ and using the properties of multiplication in $\mathcal{A}$ gives the equivalent form

(18) $$\widehat{c}_t(\bar{S}) * \widehat{c}_t(\bar{Q}) * \big(1 + p_a(t)\big) = 1\,.$$

We now note that the *harmonic number generating function*

$$\sum_{i=0}^{\infty} \mathcal{H}_i t^i = \frac{t}{1-t} + \frac{1}{2}\frac{t^2}{1-t} + \frac{1}{3}\frac{t^3}{1-t} + \cdots = \frac{\log(1-t)}{t-1}\,.$$

It follows that

$$p_a(-t) = \sum_{i=0}^{\infty} \mathcal{H}_i p_i(y) t^{i+1} = t\sum_{j=1}^{s}\sum_{i=0}^{\infty} \mathcal{H}_i (y_j t)^i = -t\sum_{j=1}^{s} \frac{\log(1-y_j t)}{1-y_j t}$$

and thus

$$p_a(t) = t\sum_{j=1}^{s} \frac{\log(1+y_j t)}{1+y_j t}\,.$$

Substituting this in equation (18) gives relation $\mathcal{R}_2$. □

Theorem 6 shows that the relations in the Arakelov Chow ring of G are the classical geometric ones perturbed by a new "arithmetic factor" of $1+p_a(t)$. While this factor is closely related to the power sums $p_i(\bar{Q})$, the most natural basis of symmetric functions for doing calculations in $CH(G)$ is the basis of Schur polynomials (corresponding to the Schubert classes; see for example [F], § 14.7). The arithmetic analogues of the special Schubert classes involve the power sum perturbation above; multiplication formulas are thus quite complicated (see [Ma]).

In geometry the Chern roots x_i and y_j all "live" on the complete flag variety above G. There are certainly natural line bundles on the flag variety whose first Chern classes correspond to the roots in Theorem 6. However on flag varieties the situation is more complicated and our knowledge is not as complete. We refer the reader to [T] for more details.

REFERENCES

[A] Arakelov, S. J. Intersection theory of divisors on an arithmetic surface. *Math. USSR Izvestiya 8* (1974), 1167–1180.

[BiGS] Bismut, J.-M., H. Gillet and C. Soulé. Analytic torsion and holomorphic determinant bundles I. Bott-Chern forms and analytic torsion. *Comm. Math. Phys. 115* (1988), 49–78.

[BC] BOTT, R. and S. S. CHERN. Hermitian vector bundles and the equidistribution of the zeroes of their holomorphic sections. *Acta. Math. 114* (1965), 71–112.

[C1] COWEN, M. J. Notes on Bott and Chern's "Hermitian vector bundles..." Preprint, Tulane University, 1973.

[C2] —— Hermitian vector bundles and value distribution for Schubert cycles. *Trans. of the A.M.S. 180* (1973), 189–228.

[D] DELIGNE, P. Le déterminant de la cohomologie, in Current Trends in Arithmetical Algebraic Geometry. *Contemp. Math. 67* (1987), 93–178.

[Do] DONALDSON, S. K. Anti-self dual Yang-Mills connections over complex algebraic surfaces and stable vector bundles. *Proc. London Math. Soc. (3) 50* (1985), 1–26.

[F] FULTON, W. *Intersection Theory*. Ergebnisse der Math. 2 (1984), Springer-Verlag.

[G] GILLET, H. Riemann-Roch theorems for higher algebraic K-theory. *Advances in Mathematics 40 No. 3* (1981), 203–289.

[GS1] GILLET, H. and C. SOULÉ. Arithmetic intersection theory. *Publ. math. I.H.E.S. 72* (1990), 94–174.

[GS2] GILLET, H. and C. SOULÉ. Characteristic classes for algebraic vector bundles with Hermitian metrics, I, II. *Annals of Math. 131* (1990), 163–203 and 205–238.

[GSZ] GILLET, H., C. SOULÉ and D. ZAGIER. Analytic torsion and the arithmetic Todd genus. *Topology 30* (1991), 21–54.

[M] MACDONALD, I. *Symmetric Functions and Hall Polynomials*. Second edition, Clarendon Press, Oxford, 1995.

[Ma] MAILLOT, V. Un calcul de Schubert arithmétique. *Duke Math. J. 80* (1995), 195–221.

[Mo] MORIWAKI, A. Inequality of Bogomolov-Gieseker type on arithmetic surfaces. *Duke Math. J. 74* (1994), 713–761.

[S] SOULÉ, C. A Vanishing theorem on arithmetic surfaces. *Invent. Math. 116* (1994), 577–599.

[SABK] SOULÉ, C., D. ABRAMOVICH, J.-F. BURNOL and J. KRAMER. *Lectures on Arakelov Geometry*. Cambridge Studies in Advanced Mathematics 33, 1992.

[Sp] SPIESS, J. Some identities involving harmonic numbers. *Math. of Computation 55* (1990), 839–863.

[T] TAMVAKIS, H. Arithmetic intersection theory on flag varieties. Preprint (1996).

(Reçu le 1er octobre 1996)

Harry Tamvakis

Department of Mathematics
University of Chicago
Chicago, IL 60637
U.S.A.