

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 43 (1997)  
**Heft:** 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** BOTT-CHERN FORMS AND ARITHMETIC INTERSECTIONS  
**Autor:** Tamvakis, Harry

**Bibliographie**  
**DOI:** <https://doi.org/10.5169/seals-63270>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 27.04.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

$$(17) \quad \widehat{c}_t(\bar{S})\widehat{c}_t(\bar{Q}) = 1 + a(\widehat{c}_t(\bar{E})) = 1 - p_a(t),$$

where the subscript  $t$  denotes the corresponding Chern polynomial. Multiplying both sides of (17) by  $1 + p_a(t)$  and using the properties of multiplication in  $\mathcal{A}$  gives the equivalent form

$$(18) \quad \widehat{c}_t(\bar{S}) * \widehat{c}_t(\bar{Q}) * (1 + p_a(t)) = 1.$$

We now note that the *harmonic number generating function*

$$\sum_{i=0}^{\infty} \mathcal{H}_i t^i = \frac{t}{1-t} + \frac{1}{2} \frac{t^2}{1-t} + \frac{1}{3} \frac{t^3}{1-t} + \cdots = \frac{\log(1-t)}{t-1}.$$

It follows that

$$p_a(-t) = \sum_{i=0}^{\infty} \mathcal{H}_i p_i(y) t^{i+1} = t \sum_{j=1}^s \sum_{i=0}^{\infty} \mathcal{H}_i (y_j t)^i = -t \sum_{j=1}^s \frac{\log(1 - y_j t)}{1 - y_j t}$$

and thus

$$p_a(t) = t \sum_{j=1}^s \frac{\log(1 + y_j t)}{1 + y_j t}.$$

Substituting this in equation (18) gives relation  $\mathcal{R}_2$ .  $\square$

Theorem 6 shows that the relations in the Arakelov Chow ring of  $G$  are the classical geometric ones perturbed by a new “arithmetic factor” of  $1 + p_a(t)$ . While this factor is closely related to the power sums  $p_i(\bar{Q})$ , the most natural basis of symmetric functions for doing calculations in  $CH(G)$  is the basis of Schur polynomials (corresponding to the Schubert classes; see for example [F], §14.7). The arithmetic analogues of the special Schubert classes involve the power sum perturbation above; multiplication formulas are thus quite complicated (see [Ma]).

In geometry the Chern roots  $x_i$  and  $y_j$  all “live” on the complete flag variety above  $G$ . There are certainly natural line bundles on the flag variety whose first Chern classes correspond to the roots in Theorem 6. However on flag varieties the situation is more complicated and our knowledge is not as complete. We refer the reader to [T] for more details.

## REFERENCES

- [A] ARAKELOV, S. J. Intersection theory of divisors on an arithmetic surface. *Math. USSR Izvestiya* 8 (1974), 1167–1180.
- [BiGS] BISMUT, J.-M., H. GILLET and C. SOULÉ. Analytic torsion and holomorphic determinant bundles I. Bott-Chern forms and analytic torsion. *Comm. Math. Phys.* 115 (1988), 49–78.

- [BC] BOTT, R. and S. S. CHERN. Hermitian vector bundles and the equidistribution of the zeroes of their holomorphic sections. *Acta. Math.* 114 (1965), 71–112.
- [C1] COWEN, M. J. Notes on Bott and Chern's "Hermitian vector bundles..." Preprint, Tulane University, 1973.
- [C2] — Hermitian vector bundles and value distribution for Schubert cycles. *Trans. of the A.M.S.* 180 (1973), 189–228.
- [D] DELIGNE, P. Le déterminant de la cohomologie, in Current Trends in Arithmetical Algebraic Geometry. *Contemp. Math.* 67 (1987), 93–178.
- [Do] DONALDSON, S. K. Anti-self dual Yang-Mills connections over complex algebraic surfaces and stable vector bundles. *Proc. London Math. Soc.* (3) 50 (1985), 1–26.
- [F] FULTON, W. *Intersection Theory*. Ergebnisse der Math. 2 (1984), Springer-Verlag.
- [G] GILLET, H. Riemann-Roch theorems for higher algebraic  $K$ -theory. *Advances in Mathematics* 40 No. 3 (1981), 203–289.
- [GS1] GILLET, H. and C. SOULÉ. Arithmetic intersection theory. *Publ. math. I.H.E.S.* 72 (1990), 94–174.
- [GS2] GILLET, H. and C. SOULÉ. Characteristic classes for algebraic vector bundles with Hermitian metrics, I, II. *Annals of Math.* 131 (1990), 163–203 and 205–238.
- [GSZ] GILLET, H., C. SOULÉ and D. ZAGIER. Analytic torsion and the arithmetic Todd genus. *Topology* 30 (1991), 21–54.
- [M] MACDONALD, I. *Symmetric Functions and Hall Polynomials*. Second edition, Clarendon Press, Oxford, 1995.
- [Ma] MAILLOT, V. Un calcul de Schubert arithmétique. *Duke Math. J.* 80 (1995), 195–221.
- [Mo] MORIWAKI, A. Inequality of Bogomolov-Gieseker type on arithmetic surfaces. *Duke Math. J.* 74 (1994), 713–761.
- [S] SOULÉ, C. A Vanishing theorem on arithmetic surfaces. *Invent. Math.* 116 (1994), 577–599.
- [SABK] SOULÉ, C., D. ABRAMOVICH, J.-F. BURNOL and J. KRAMER. *Lectures on Arakelov Geometry*. Cambridge Studies in Advanced Mathematics 33, 1992.
- [Sp] SPIESS, J. Some identities involving harmonic numbers. *Math. of Computation* 55 (1990), 839–863.
- [T] TAMVAKIS, H. Arithmetic intersection theory on flag varieties. Preprint (1996).

(Reçu le 1<sup>er</sup> octobre 1996)

Harry Tamvakis

Department of Mathematics  
University of Chicago  
Chicago, IL 60637  
U.S.A.