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$H_1(G, \mathbf{Q})$. We can find an independent subsystem $\{\bar{x}_{i_1}, \dots, \bar{x}_{i_{m-1}}\}$ in $H_1(G, \mathbf{Q})$ such that its pre-image $\{x_{i_1}, \dots, x_{i_m}\}$ freely generates a free group. Therefore $\lambda_X(G) \geq 2(m-1) - 1 = 2m - 3$. \square

It seems to us that for a one-relator group G of rank $m \geq 3$ the inequality $\lambda_*(G) \geq 2m - 3$ cannot be deduced directly from Magnus' Theorem as it is claimed in [GrLP].

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