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EVEN NON-SPIN MANIFOLDS, SPIN^c STRUCTURES, AND DUALITY

by Daniel ACOSTA and Terry LAWSON

ABSTRACT. This note explores the restrictions on the second Stiefel-Whitney class w_2 of a smooth closed oriented 4-manifold which has an even intersection form but is not spin. The Hom dual of w_2 is shown to be non-integral, whereas the existence of a spin^c structure means that its Poincaré dual is the reduction of an integral class. We examine this in detail in a simple example $S^2 \times S^2/\{\pm 1\}$.

In [H, p. 23] N. Habegger gives $M = S^2 \times S^2/(x,y) \sim (-x,-y)$ as an example of an oriented, non-spin smooth 4-manifold with an even intersection form. In discussing this example in [K, p. 27] R. Kirby seems to be relating it to the (non-)integrality of the second Stiefel-Whitney class $w_2(M)$. However, for a closed, oriented, smooth 4-manifold X, it is always the case that the second Stiefel-Whitney class $w_2(X)$ is the mod 2 reduction of an integral class. This was first shown by Hirzebruch and Hopf in [HH, p. 169], and is a key step in showing that X admits a spin^c structure. Spin^c structures have recently become very important as they are involved in the Seiberg-Witten invariants, now a major area of study in the topology and geometry of 4-manifolds (see, e.g., [W], [T], [KM]).

In this expository paper we want to explore some of the interesting phenomena at work in this example and describe the characterizing property which w_2 possesses. On the way we shall encounter many important concepts in geometric topology, including Poincaré and Hom duality, the intersection form, even forms, spin structures, spin^c structures, and characteristic classes. The article is intended for readers who have a background of a year-long graduate course in topology from a text such as Bredon [B].

We start by reviewing some basic definitions. We will assume X is a compact, oriented smooth 4-manifold. When the coefficient group is not

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