

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 43 (1997)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ON CYCLOTOMIC POLYNOMIALS, POWER RESIDUES, AND RECIPROCITY LAWS
Autor: Sharifi, Romyar T.
Kapitel: 5. Homogeneous polynomials
DOI: <https://doi.org/10.5169/seals-63283>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 13.03.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

5. HOMOGENEOUS POLYNOMIALS

Generalizing Theorem 1 to include homogeneous polynomials introduces subtle difficulties, which we address in the following proof.

Proof of Theorem 2. Let K and L be as in the proof of Theorem 1. We set $\pi_n = y - qx\zeta_n$ and $\pi = N_{L/K}(\pi_n)$ and note $p = N_K(\pi)$ with the assumption $s > 0$. Now let a be an integer dividing x , and decompose a as $a = a'q^k$ where a' is not divisible by q . Let $y' = N_{L/K}(y) = y^{[L:K]}$. We remark that $\pi \equiv y' \pmod{qa}$ since $\pi_n \equiv y \pmod{qa}$.

We can now apply reciprocity. We are interested in evaluating the symbol $(a/\pi)_l$. We can use reciprocity laws (5) and (6) along with multiplicativity from Theorems 5(a) and 6(a) in the following manner. We have

$$\left(\frac{a}{\pi}\right)_l = \left(\frac{a'}{\pi}\right)_l \left(\frac{q}{\pi}\right)_l^k = \left(\frac{\pi}{a'}\right)_l (\pi, a')_l (\pi, q)_l^k = \left(\frac{\pi}{a'}\right)_l (\pi, a)_l.$$

Now $\pi \equiv y' \pmod{a'}$ so $(\pi/a')_l = (y'/a')_l$ by Theorem 5(c). And letting $(y')^{-1}$ denote the q -adic inverse of y' in \mathbf{Z}_q , we have

$$(\pi, a)_l = (\pi(y')^{-1}, a)_l (y', a)_l.$$

Let $\pi' = \pi(y')^{-1}$. Now $\pi' \equiv 1 \pmod{qa}$. So $\pi' \equiv 1 \pmod{q}$, and if q divides a , then $\pi' \equiv 1 \pmod{q^2}$. Thus the fact that λ_q^2 divides q implies $\pi' \equiv 1 \pmod{f_l(a)}$ by Corollary 8. Thus $(\pi', a)_l = 1$.

We now have that

$$\left(\frac{a}{\pi}\right)_l = \left(\frac{y'}{a'}\right)_l (y', a)_l.$$

The symbol $(y'/a')_l$ is an l -th root of unity, and by Theorem 5(d) an element $\sigma \in G_K$ acts on it as follows:

$$\sigma\left(\frac{y'}{a'}\right)_l = \left(\frac{\sigma y'}{\sigma a'}\right)_l = \left(\frac{y'}{a'}\right)_l,$$

since a' and y' are rational integers. Since l is odd, the only such root of unity fixed under the action of the Galois group is 1. In the same manner, Theorem 6(h) enables us to see that $(y', a)_l = 1$. We therefore conclude that $(a/\pi)_l = 1$. \square