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THEOREM 8. *Let X be a Kählerian threefold and $h \in H^2(X, \mathbf{R})$ be a Kähler class. Then the map*

$$\begin{array}{rccc} \langle .,. \rangle : & H^2(X,\mathbf{R}) \times H^2(X,\mathbf{R}) & \longrightarrow & \mathbf{R} \\ & (a,b) & \longmapsto & a \cup b \cup h \end{array}$$

is a non-degenerate, symmetric bilinear form of signature $(2h^{2,0}+1, h^{1,1}-1)$.

One can restate this theorem in such a form as to obtain – at least in theory – some explicit inequalities in the coefficients of cubic polynomials which are satisfied by the cup forms of Kählerian and hence projective algebraic threefolds. The main result of [Sch2] is

THEOREM 9. *For $n \geq 4$, the polynomial*

$$x_0 \left(\frac{4-n}{4} x_0^2 - 3x_1^2 - \cdots - 3x_n^2 \right)$$

cannot occur as the (real) cup form of a projective algebraic threefold with $b_1 = 0$ and $b_3 = 0$.

As a corollary, one obtains the following generalization of a result of Campana and Peternell [CP]:

THEOREM 10. *For $n \geq 4$, twistor spaces over $\sharp_{i=1}^n \mathbf{P}_2$ are not homeomorphic to projective algebraic threefolds.*

REFERENCES

[AGV] ARNOLD, V.I., S.M. GUSEIN-ZADE and A.N. VARCHENKO. *Singularities of Differentiable Maps, Vol. 1.* Birkhäuser, 1985.

[BC] BARDELLI, F. and A. DEL CENTINA. Nodal cubic surfaces and the rationality of the moduli space of curves of genus two. *Math. Ann. 270* (1985), 599–602.

[Be] BEKLEMISHEV, N.D. Invariants of cubic forms in four variables. *Vestnik Mosk. Univ. Mat. 37* (1982), 42–9.

[BW] BRUCE, J.W. and C.T.C. WALL. On the classification of cubic surfaces. *J. London Math. Soc. (2) 19* (1979), 245–56.

[CP] CAMPANA, F. and T. PETERNELL. Rigidity of Fano 3-folds. *Comm. Anal. Geom. 2* (1994), 173–201.

[GH] GRIFFITHS, Ph. and J. HARRIS. *Principles of Algebraic Geometry.* Wiley Interscience, 1978.

[Ha1] HARTSHORNE, R. Generalized divisors on Gorenstein schemes. *K-Theory 8* (1995), 287–339.

[Ha2] —— Families of curves in $\mathbf{P}^3$ and Zeuthen's problem. Preprint, Berkeley.

[Hi] HILBERT, D. Über die vollen Invariantensysteme. *Math. Ann. 42* (1893), 313–73.

[Is] ISHII, S. Moduli space of polarized Del Pezzo surfaces and its compactification. *Tokyo J. Math. 5* (1982), 289–97.

[Ju] JUPP, P. E. Classification of certain 6-manifolds. *Proc. Cambridge Phil. Soc. 36* (1973), 293–300.

[La] LAMOTKE, K. The topology of complex projective varieties after S. Lefschetz. *Topology 20* (1981), 15–51.

[Ne] NEWSTEAD, P. E. *Introduction to Moduli Problems and Orbit Spaces.* Springer, 1978.

[OV] OKONEK, Ch. and A. VAN DE VEN. Cubic forms and complex threefolds. *L'Enseignement Math. 41* (1995), 297–333.

[Sch1] SCHMITT, A. Zur Topologie dreidimensionaler komplexer Mannigfaltigkeiten. Ph. D. thesis. Zürich, 1995.

[Sch2] —— On the non-existence of Kähler structures on certain closed and oriented 6-manifolds. *J. reine angew. Math. 479* (1996), 205–216.

[Sch3] —— On the classification of certain 6-manifolds and applications to algebraic geometry. *Topology 36* (1997), 1291–1315.

[Se] SEGRE, B. *The Nonsingular Cubic Surfaces.* Oxford at the Clarendon Press, 1942.

[St] STURMFELS, B. *Algorithms in Invariant Theory.* Springer, 1993.

[Wa] WALL, C. T. C. Classification problems in differential topology V. On certain 6-manifolds. *Inventiones Math. 1* (1966), 335–74.

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