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(7.5) A space of great interest nowadays is the moduli space of flat $SU(2)$ connections on a punctured Riemann sphere — in the language of this paper, geodesic polygons in S^3 (rather than \mathbf{R}^3). The spaces here can be seen as limiting versions where the radius of S^3 goes to infinity. We do not know how to adapt the Gel'fand-MacPherson correspondence to this case; one definite complication is that it is no longer the symmetric group but the braid group which permutes the edges, and that action is not complex.

(7.6) By averaging the Riemannian metric with respect to the bending torus, one can deform the complex structure on a space of prodigal polygons to that of the corresponding toric variety. Is the original complex structure that of a toric variety (not just in the same deformation class) ?

REFERENCES

- [Au] AUDIN, M. *The topology of torus actions on symplectic manifolds*. Birkhäuser, 1991.
- [DJ] DAVIS, M. and T. JANUSZKIEWICZ. Convex polytopes, Coxeter orbifolds and torus actions. *Duke Math. J.* 62 (1991), 417–451.
- [De] DELZANT, T. Hamiltoniens périodiques et image convexe de l'application moment. *Bull. Soc. Math. France* 116 (1988), 315–339.
- [Du] DUISTERMAAT, J. J. Convexity and tightness for restrictions of Hamiltonian functions to fixed-point sets of an antisymplectic involution. *Trans. AMS* 275 (1983), 417–429.
- [Fr] FRANKLIN, J. *Matrix theory*. Prentice-Hall, 1968.
- [GGMS] GEL'FAND, I., M. GORESKY, R. MACPHERSON and V. SERANOVA. Combinatorial geometries, convex polyedra and Schubert cells. *Adv. Math.* 63 (1987), 301–316.
- [GM] GEL'FAND, I. and R. MACPHERSON. Geometry in Grassmannians and a generalization of the dilogarithm. *Adv. Math.* 44 (1982), 279–312.
- [Gu] GUILLEMIN, V. *Moment maps and combinatorial invariants of Hamiltonian T^n -spaces*. Birkhäuser, 1994.
- [GS1] GUILLEMIN, V. and S. STERNBERG. The Gelfand-Cetlin system and quantization of the complex flag manifolds. *J. Funct. Anal.* 52(1) (1983), 106–128.
- [GS2] GUILLEMIN, V. and S. STERNBERG. Birational equivalence in the symplectic category. *Inventiones Math.* 97 (1989), 485–522.
- [Ha] HAUSMANN, J.-C. Sur la topologie des bras articulés. In “Algebraic Topology, Poznan”. *Springer Lectures Notes* 1474 (1989), 146–159.
- [HK] HAUSMANN, J.-C. and A. KNUTSON. The cohomology ring of polygon spaces. Preprint (1997). <http://www.unige.ch/math/biblio>
- [Ho] HOPF, H. Über die Abbildungen der dreidimensionalen Sphäre auf die Kugelfläche. *Math. Annalen* 104 (1931), 637–665.

- [KM1] KAPOVICH, M. and J. MILLSON. The symplectic geometry of polygons in the Euclidean plane. *J. Diff. Geometry* 42 (1995), 430–464.
- [KM2] KAPOVICH, M. and J. MILLSON. The symplectic geometry of polygons in Euclidean space. *J. Diff. Geometry* 44 (1996), 479–513.
- [Ki] KIRWAN, F. *Cohomology of quotients in symplectic and algebraic geometry*. Princeton University Press, 1984.
- [Kl] KLYACHKO, A. Spatial polygons and stable configurations of points in the projective line, in : *Algebraic geometry and its applications (Yaroslavl, 1992)*, *Aspects Math.*, Vieweg, Braunschweig (1994), 67–84.
- [Mo] MONTGOMERY, R. Heisenberg and isoholonomic inequalities : “How to use dual pairs to ignore the rest of the universe”. *Symplectic geometry and mathematical physics*. Souriau Volume. Birkhäuser, 1991.
- [Th] THIMM, A. Integrable geodesic flows on homogeneous spaces. *Ergodic theory and dynamical systems* 1 (1980), 495–517.
- [TW] THURSTON, W. and J. WEEKS. The mathematics of three-dimensional manifolds. *Scientific American* 251 (1984), 94–107.
- [Wa] WALKER, K. Configuration spaces of linkages. Undergr. Thesis, Princeton (1985).

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