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(7.5) A space of great interest nowadays is the moduli space of flat  $SU(2)$  connections on a punctured Riemann sphere — in the language of this paper, geodesic polygons in  $S^3$  (rather than  $\mathbf{R}^3$ ). The spaces here can be seen as limiting versions where the radius of  $S^3$  goes to infinity. We do not know how to adapt the Gel'fand-MacPherson correspondence to this case; one definite complication is that it is no longer the symmetric group but the braid group which permutes the edges, and that action is not complex.

(7.6) By averaging the Riemannian metric with respect to the bending torus, one can deform the complex structure on a space of prodigal polygons to that of the corresponding toric variety. Is the original complex structure that of a toric variety (not just in the same deformation class)?

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