

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 42 (1996)
Heft: 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: SQUARE-FREE TOWER OF HANOI SEQUENCES
Autor: HINZ, Andreas M.
Kurzfassung
DOI: <https://doi.org/10.5169/seals-87878>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 19.08.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

SQUARE-FREE TOWER OF HANOI SEQUENCES

by Andreas M. HINZ

ABSTRACT. The sequence of moves in the optimal solution of the Tower of Hanoi with an arbitrary number of discs has recently been shown to lead to an example of an infinite square-free string over a six-letter alphabet by recourse to the theory of iterated morphisms. We present a direct approach to this result, using only properties of the Tower of Hanoi itself, which also reveals an implicit infinite square-free string with just five letters.

0. SQUARE-FREE STRINGS AND THE OLIVE SEQUENCE

Suppose you dispose of a reservoir of *letters* (or *symbols*), i.e. an at most countably infinite set \mathcal{A} , called an *alphabet*, and you are asked to construct a *word* (or *string*) of infinite length, i.e. a sequence $a \in \mathcal{A}^{\mathbb{N}}$, which does not contain any non-trivial immediate repetition, or *square*, i.e. there are no $m \in \mathbb{N}_0$ and $l \in \mathbb{N}$ such that

$$\forall k \in \{m+1, \dots, m+l\} : a_{k+l} = a_k .$$

(For a concise survey on *square-free* words and their use in mathematics see [4].)

Assume $\mathcal{A} = \mathbb{N}$ and try an apparently economic approach, namely choose a_k as the smallest positive integer such that (a_1, \dots, a_k) does not contain any square. Then you come up with the following sequence :

$$(1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1, 5, 1, 2, 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1, \dots)$$

A connoisseur of the mathematical theory of the Tower of Hanoi (TH) puzzle (see [13] for a comprehensive survey) will immediately recognize the pattern