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THEOREM 7.3. $T \cong PIP^+(S^1)$.

The three functions in $PIP^+(S^1)$ corresponding to A , B , and C are the following.

$$A(t) = \begin{cases} \frac{t}{t+1}, & 0 \leq t \leq \frac{1}{2} \\ \frac{-t+1}{-5t+4}, & \frac{1}{2} \leq t \leq \frac{2}{3} \\ \frac{2t-1}{t}, & \frac{2}{3} \leq t \leq 1 \end{cases} \quad B(t) = \begin{cases} t, & 0 \leq t \leq \frac{1}{2} \\ \frac{3t-1}{4t-1}, & \frac{1}{2} \leq t \leq \frac{2}{3} \\ \frac{-6t+5}{-11t+9}, & \frac{2}{3} \leq t \leq \frac{3}{4} \\ \frac{2t-1}{t}, & \frac{3}{4} \leq t \leq 1 \end{cases}$$

$$C(t) = \begin{cases} \frac{-3t+2}{-5t+3}, & 0 \leq t \leq \frac{1}{2} \\ \frac{2t-1}{t}, & \frac{1}{2} \leq t \leq \frac{2}{3} \\ \frac{5t-3}{7t-4}, & \frac{2}{3} \leq t \leq 1 \end{cases}$$

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