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## CENTRALISERS IN THE BRAID GROUP AND SINGULAR BRAID MONOID

by Roger FENN, Dale ROLFSEN and Jun ZHU<sup>1)</sup>

ABSTRACT. The centre of the braid group  $B_n$  is well-known to be infinite cyclic and generated by a twist braid. In this paper we consider the centraliser of certain important subgroups, and in particular we characterise the elements of  $B_n$  which commute with one of the usual generators  $\sigma_j$ . This characterisation is generalised to the monoid of singular braids  $SB_n$ , recently introduced (independently) by J. Baez and J. Birman. We determine the singular braids which commute with  $\sigma_j$ , or with a singular generator  $\tau_j$ ; in fact we show these submonoids are the same.

We establish that the centraliser in  $B_n$  of  $\sigma_j$  is isomorphic to the cartesian product of two groups: the group of  $(n-1)$ -braids whose permutations stabilise  $j$  and the group of integers. More generally, we show that the centraliser of the naturally-included braid subgroup  $B_r \subset B_n$  likewise splits as a direct product, and we give an explicit presentation for this centraliser. We also describe the centralisers of  $SB_r \subset SB_n$ .

As another application we consider a conjecture of J. Birman regarding the injectivity of a map, related to Vassiliev theory,  $\eta: SB_n \rightarrow \mathbf{Z}B_n$  from the singular braid monoid to the group ring of the braid group. We see that the question is related to the centraliser problem and prove the injectivity of  $\eta$  for braids with up to two singularities.

### 1. INTRODUCTION AND BASIC DEFINITIONS

The braid group  $B_n$ , for an integer  $n \geq 2$ , may be considered abstractly as the group with generators  $\sigma_1, \dots, \sigma_{n-1}$  and relations

$$\begin{aligned} \sigma_j \sigma_k &= \sigma_k \sigma_j & \text{if } |j - k| > 1, \\ \sigma_j \sigma_k \sigma_j &= \sigma_k \sigma_j \sigma_k & \text{if } |j - k| = 1. \end{aligned}$$

There are equivalent geometric descriptions of braids as strings in space, as automorphisms of a free group  $F_n$ , as the fundamental group of a configuration space, or as homeomorphisms of an  $n$ -punctured plane (see below), which explains the importance of the braid groups in many

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