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CENTRALISERS IN THE BRAID GROUP AND SINGULAR BRAID MONOID

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ABSTRACT. The centre of the braid group B_n is well-known to be infinite cyclic and generated by a twist braid. In this paper we consider the centraliser of certain important subgroups, and in particular we characterise the elements of B_n which commute with one of the usual generators σ_j. This characterisation is generalised to the monoid of singular braids SB_n, recently introduced (independently) by J. Baez and J. Birman. We determine the singular braids which commute with σ_j, or with a singular generator τ_j; in fact we show these submonoids are the same.

We establish that the centraliser in B_n of σ_j is isomorphic to the cartesian product of two groups: the group of $(n-1)$-braids whose permutations stabilise j and the group of integers. More generally, we show that the centraliser of the naturally-included braid subgroup $B_r \subset B_n$ likewise splits as a direct product, and we give an explicit presentation for this centraliser. We also describe the centralisers of $SB_r \subset SB_n$.

As another application we consider a conjecture of J. Birman regarding the injectivity of a map, related to Vassiliev theory, $\eta : SB_n \to \mathbf{Z}B_n$ from the singular braid monoid to the group ring of the braid group. We see that the question is related to the centraliser problem and prove the injectivity of η for braids with up to two singularities.

1. INTRODUCTION AND BASIC DEFINITIONS

The braid group B_n, for an integer $n \geqslant 2$, may be considered abstractly as the group with generators $\sigma_1, \ldots, \sigma_{n-1}$ and relations

$$\sigma_j \sigma_k = \sigma_k \sigma_j \qquad \text{if} \quad |j-k| > 1,$$
$$\sigma_j \sigma_k \sigma_j = \sigma_k \sigma_j \sigma_k \qquad \text{if} \quad |j-k| = 1 .$$

There are equivalent geometric descriptions of braids as strings in space, as automorphisms of a free group F_n, as the fundamental group of a configuration space, or as homeomorphisms of an n-punctured plane (see below), which explains the importance of the braid groups in many

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