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Autor:	Fenn, Roger / Rourke, Colin
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where the maps α, β, γ and δ are induced by inclusion. In order for γ to be a well defined homomorphism it is necessary to check that the relation $w^{-1}hw = h^\phi$, $h \in H$ is a consequence of the relations $t^{-1}ht = h^\phi$, $[a, t^{-1}w] = 1$, $[t, w] = 1$, $h \in H$, $a \in A$. But this follows because $w^{-1}hw = w^{-1}tt^{-1}htt^{-1}w = w^{-1}th^\phi t^{-1}w = h^\phi$. Now α is injective because A' is an HNN extension of A (see [DD, p. 33] or [Se, p. 9]) and β is injective because of theorem 6.1. So δ is injective and this proves the theorem. \square

THEOREM 6.3. *Let*

$$(*) \quad u_i(t) = 1, i \in I$$

*be a set of equations over the group A where the exponent sum of t in each $u_i(t)$ is zero. Suppose $w = w(t) \in A * \langle t \rangle - A$ and the factors of w are all torsion free. Then the set of equations*

$$(**) \quad u_i(w(t)) = 1, i \in I$$

has a solution over A if and only if the set $()$ has a solution over A .*

Proof. Let $w(t) = at$ where $a \in A$ has infinite order. Then a solution x for $u_i(w(t)) = 1$ defines a solution at for $(*)$.

Conversely suppose $x \in A'$ is a solution of the set of equations $\{u_i(t) = 1 \mid i \in I\}$. Let G be the subgroup of A' generated by

$$\{x^{-n}ax^n \mid a \in A, n \in \mathbf{Z}\}.$$

Then A is a subgroup of G and G is a subgroup of

$$H = \langle G, t \mid w^{-1}gw = g^\phi, g \in G \rangle$$

where $g^\phi = x^{-1}gx$ by theorem 6.2. Because of the exponent sum condition $u_i(w) = 1, i \in I$. \square

REFERENCES

- [BRS] BUONCHRISTIANO, S., B. J. SANDERSON and C. P. ROURKE. *A geometric approach to homology theory, VII: the geometry of CW complexes.* London Maths. Soc. Lecture Note Series 18, 131-149, C.U.P. (1976).
- [DD] DICKS, W. and M. DUNWOODY. *Groups acting on Graphs.* Cambridge Studies in Advanced Maths. 17, C.U.P. (1983).
- [EH] EDJVET, M. and J. HOWIE. The solution of length four equations over groups. *Trans. Amer. Math. Soc.* 326 (1991), 345-369.
- [F] FENN, R. A. *Techniques of Geometric Topology.* London Maths. Soc. Lecture Note Series 57, C.U.P. (1983).

- [GR] GERSTENHABER, M. and O.S. ROTHAUS. The Solution of Sets of Equations in Groups. *Proc. Nat. Acad. Sci. USA* 48 (1962), 1531-1533.
- [H₁] HOWIE, J. The solutions of length three equations over groups. *Proc. Edinburgh Maths. Soc.* 26 (1983), 89-96.
- [H₂] —— On pairs of 2-complexes and systems of equations over groups. *J. Reine. Angew. Math.* 324 (1981), 165-174.
- [Ke] KERVAIRE, M. On higher dimensional knots. *Differential and combinatorial topology — a symposium in honour of Marston Morse*. Princeton Math. Series 27 (1965).
- [Kl] KLYACHKO, A. Funny property of sphere and equations over groups. *Comm. in Alg.* 21 (7) (1993), 2555-2575.
- [L] LEVIN, F. Solutions of equations over groups. *Bull. Amer. Math. Soc.* 68 (1962), 603-604.
- [MKS] MAGNUS, W., A. KARRAS and D. SOLITAR. *Combinatorial Group Theory*. Interscience (1966).
- [N] NEUMANN, B. H. Adjunction of elements to groups. *J. London Math. Soc.* 18 (1943), 4-11.
- [Ro] ROTHAUS, O.S. On the non-triviality of some group extensions given by generators and relations. *Ann. of Math.* (2) 106 (1977), 599-612.
- [R₁] ROURKE, C. P. Presentations and the trivial group. Proceedings of the 1977 Sussex Topology Conference, Springer lecture notes, 722, 134-143.
- [R₂] —— On dunce hats and the Kervaire conjecture. Papers presented to Christopher Zeeman, University of Warwick, 221-230 (1988).
- [Se] SERRE, J.-P. *Trees*. Springer Verlag (1980).
- [Sh] SHORT, H. Topological Methods in Group Theory: the Adjunction problem. Ph.D. thesis Warwick University (1979).

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Roger Fenn

Department of Mathematics
Sussex University
Falmer, Brighton BN1 9QH
England

Colin Rourke

Mathematics Institute
University of Warwick
Coventry CV4 7AL
England