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OVER TORSION-FREE GROUPS  
**Autor:** Fenn, Roger / Rourke, Colin

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where the maps  $\alpha, \beta, \gamma$  and  $\delta$  are induced by inclusion. In order for  $\gamma$  to be a well defined homomorphism it is necessary to check that the relation  $w^{-1}hw = h^\phi, h \in H$  is a consequence of the relations  $t^{-1}ht = h^\phi, [a, t^{-1}w] = 1, [t, w] = 1, h \in H, a \in A$ . But this follows because  $w^{-1}hw = w^{-1}tt^{-1}htt^{-1}w = w^{-1}th^\phi t^{-1}w = h^\phi$ . Now  $\alpha$  is injective because  $A'$  is an HNN extension of  $A$  (see [DD, p. 33] or [Se, p. 9]) and  $\beta$  is injective because of theorem 6.1. So  $\delta$  is injective and this proves the theorem.  $\square$

THEOREM 6.3. *Let*

$$(*) \quad u_i(t) = 1, i \in I$$

*be a set of equations over the group  $A$  where the exponent sum of  $t$  in each  $u_i(t)$  is zero. Suppose  $w = w(t) \in A * \langle t \rangle - A$  and the factors of  $w$  are all torsion free. Then the set of equations*

$$(**) \quad u_i(w(t)) = 1, i \in I$$

*has a solution over  $A$  if and only if the set (\*) has a solution over  $A$ .*

*Proof.* Let  $w(t) = at$  where  $a \in A$  has infinite order. Then a solution  $x$  for  $u_i(w(t)) = 1$  defines a solution  $at$  for (\*).

Conversely suppose  $x \in A'$  is a solution of the set of equations  $\{u_i(t) = 1 \mid i \in I\}$ . Let  $G$  be the subgroup of  $A'$  generated by

$$\{x^{-n}ax^n \mid a \in A, n \in \mathbf{Z}\}.$$

Then  $A$  is a subgroup of  $G$  and  $G$  is a subgroup of

$$H = \langle G, t \mid w^{-1}gw = g^\phi, g \in G \rangle$$

where  $g^\phi = x^{-1}gx$  by theorem 6.2. Because of the exponent sum condition  $u_i(w) = 1, i \in I$ .  $\square$

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Roger Fenn

Department of Mathematics  
Sussex University  
Falmer, Brighton BN1 9QH  
England

Colin Rourke

Mathematics Institute  
University of Warwick  
Coventry CV4 7AL  
England