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through the Committee for Undergraduate Mathematics Education (CRUME) was also rejected.

Writing recently in the Bulletin of the American Mathematical Society, Thurston remarked :

Mathematicians have developed habits of communication that are often dysfunctional.

and he went on to intimate how so many mathematicians fail to communicate in research colloquia through using highly technical language without explanation or motivation for non-experts. He continued by noting a similar problem in teaching :

...in classrooms... we go through the motions of saying for the record what we think the student “ought” to learn, while the students are trying to grapple with the more fundamental issues of learning our language and guessing at our mental models. Books compensate by giving samples of how to solve every type of homework problem. Professors compensate by giving homework and tests that are much easier than the material “covered” in the course, and then grading the homework and tests on a scale that requires little understanding. We assume that the problem is with the students rather than with the communication: that the students either just don’t have what it takes, or else just don’t care. Outsiders are amazed at this phenomenon, but within the mathematical community, we dismiss it with shrugs.

(Thurston, 1994, p. 166)

I cannot believe that mathematicians can continue to ignore the study of mathematical thinking as part of the totality of the profession, for if it is not done by mathematicians, others surely lack the mathematical knowledge to research it in depth. I suggest that the study of mathematical thinking be given a place in the canons of mathematical activity comparable with other areas of mathematics. Just as a topologist will defend a number-theorist’s right to do research within the umbrella of mathematics I hope that specialists in mathematical research will similarly defend the right of mathematicians to do research into mathematical thinking. Respect will have to be earned by mathematics educators. But if opportunities to earn respect are not honoured then mathematics itself can only be the poorer.

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