But a considerable minority (14 out of 44, that is 32%) reported that the mathematics they were being taught did not allow them to think in a problem-solving manner:

> Since following the course I know mathematics is about solving problems. But whatever mathematics I am doing now doesn't allow me to do all those things. They are just more things to be remembered.
>
> *(Male computer education student)*

> I believed mathematics is useful in that it helps me to think. Having said that it is hard to say how I can do this with the maths I am doing. Most of the questions given can be solved by applying directly the procedures we had just learned. There is nothing to think about.
>
> *(Female industrial science student majoring in mathematics)*

So what does this tell us? One interpretation may be that the problem-solving had relatively easy problems that allowed the students to "think mathematically" but that "serious" mathematics is demanding.

Evidence from another source suggests that more open methods can work in analysis courses. In an experiment in Grenoble, large classes of analysis students were encouraged to work in groups in the lecture hall to propose theorems which they and other students subjected to a process of either proof or refutation by counter-example. A small minority said they preferred being told how to do mathematics in lecture classes, but 80% said they preferred the exploratory form of learning (Alibert, 1988; Alibert & Thomas, 1991).

It seems to me more likely that, because we fear failure in our students, we resort to the methods that "seem" necessary throughout mathematics. When students are likely to fail, we lack the faith in their ability to think for themselves and *tell* them how to do the mathematics in an organised way. The result is that they behave as we expect, rather than as we might prefer — they learn the material to pass the exam.

Reflections on mathematical thinking

Currently the university mathematics community is under some stress because it earns part of its finance from teaching undergraduates and all is not well. In the UK the London Mathematical Society produced a report which changed the British undergraduate degree structure to allow for four years instead of the traditional three. Yet when I asked the LMS to change my area of research interest to "Advanced Mathematical Thinking", the committee reluctantly refused because it was not an accepted heading in the American Mathematical Society's listing of topics. A formal request passed to the AMS

through the Committee for Undergraduate Mathematics Education (CRUME) was also rejected.

Writing recently in the Bulletin of the American Mathematical Society, Thurston remarked :

> Mathematicians have developed habits of communication that are often dysfunctional.

and he went on to intimate how so many mathematicians fail to communicate in research colloquia through using highly technical language without explanation or motivation for non-experts. He continued by noting a similar problem in teaching :

> ... in classrooms... we go through the motions of saying for the record what we think the student "ought" to learn, while the students are trying to grapple with the more fundamental issues of learning our language and guessing at our mental models. Books compensate by giving samples of how to solve every type of homework problem. Professors compensate by giving homework and tests that are much easier than the material "covered" in the course, and then grading the homework and tests on a scale that requires little understanding. We assume that the problem is with the students rather than with the communication : that the students either just don't have what it takes, or else just don't care. Outsiders are amazed at this phenomenon, but within the mathematical community, we dismiss it with shrugs.
>
> *(Thurston, 1994, p. 166)*

I cannot believe that mathematicians can continue to ignore the study of mathematical thinking as part of the totality of the profession, for if it is not done by mathematicians, others surely lack the mathematical knowledge to research it in depth. I suggest that the study of mathematical thinking be given a place in the canons of mathematical activity comparable with other areas of mathematics. Just as a topologist will defend a number-theorist's right to do research within the umbrella of mathematics I hope that specialists in mathematical research will similarly defend the right of mathematicians to do research into mathematical thinking. Respect will have to be earned by mathematics educators. But if opportunities to earn respect are not honoured then mathematics itself can only be the poorer.

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