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FACTOR EQUIVALENCE RESULTS FOR INTEGERS AND UNITS

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ABSTRACT. We give alternative proofs of two results of Fröhlich on the Galois module structure of the ring of integers and of the group of S-units in a Galois extension of number fields. We also point out applications to index computations in rings of integers and to class number relations.

1. INTRODUCTION

The purpose of this note is to give a brief presentation of basic factor equivalence results about the Galois module structure of the ring of integers and of the group of units in a Galois extension of number fields. Such results were first given by Nelson [12] and by Fröhlich [8, 9]. In [8] and [4, § 3] these results are proved for abelian and for "admissible" Galois groups. It was shown later by Ritter and Weiss that all finite groups are "admissible" [14]. The proofs given below do not use any subtle representation-theoretic properties such as admissibility.

We set up the terminology in the next section. In Section 3 we show that the ring of integers in a Galois extension of number fields is "factor equivalent" to the group ring of the Galois group over the ring of integers of the base field. The proof uses the conductor discriminant formula, and it holds in the more general context of extensions of Dedekind domains of characteristic zero with separable residue field extensions.

In Section 4 the factor equivalence class of the lattice of units is expressed in terms of class numbers of intermediate fields. The proof uses zeta-functions and it holds for arbitrary Galois extensions of number fields.

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