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HIGHER EULER CHARACTERISTICS (I)

by Ross GEOGHEGAN[1]) and Andrew NICAS[2])

To Peter Hilton on the occasion of his 70-th birthday.

ABSTRACT. The classical Euler characteristic $\chi \equiv \chi_0$ of a finite complex lies at the bottom of a sequence of homotopy invariants. The next invariant in this sequence χ_1 is introduced here and studied in some detail. The rest of the sequence, χ_n with $n \geqslant 2$, will be discussed in a sequel paper. Applications to geometric group theory are found by considering the behavior of χ_1 on an aspherical finite complex of fundamental group G. Just as the $\chi(G) \neq 0$ implies that the center of G is trivial (Gottlieb's Theorem), it is shown here that (under a weak additional hypothesis and using rational coefficients) $\chi_1(G) \neq 0$ implies that the center of G is infinite cyclic. We also find a generalization of Gottlieb's Theorem in which the Lefschetz number of an automorphism of G is related to the fixed subgroup of the automorphism.

INTRODUCTION

From our point of view, the classical Euler characteristic of a finite complex is "zero-th order". In this paper we introduce a "first order" analog, a new invariant in topology and group theory. In a sequel paper and in [GNO] we extend these ideas to an "n-th order" Euler characteristic for all positive n.

For a finite complex X, the new invariant $\chi_1(X; R)$, defined in § 1, comes in different forms, depending on the coefficient ring R; and a more sophisticated version $\tilde{\chi}_1(X; R)$ defined in § 2, involves the universal cover of X. By contrast, the classical analogs of these are essentially the same, namely the integer $\chi(X)$. We should tell the reader from the start that all our first order invariants are trivial if X is simply connected.

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