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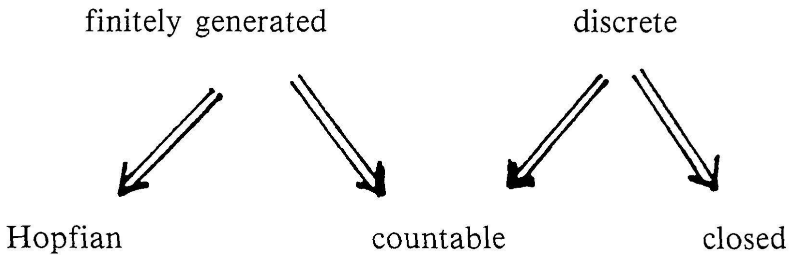
Another general question is:

QUESTION 3. *What residually finite groups have a faithful chaotic action on some smooth connected compact manifold?*

Clearly finite groups are residually finite but have no faithful chaotic actions on any connected compact manifold. On the other hand, if a group G acts faithfully and chaotically on a compact manifold, then is G necessarily countable?, finitely generated?, discrete as a subgroup of $\mathrm{Hom}(M)$?, closed as a subgroup of $\mathrm{Hom}(M)$? These properties hold for the known examples of chaotic actions constructed from the action of $SL(n, \mathbf{Z})$ on $\mathbf{T}^n$. The properties would seem unlikely to hold in general, but counterexamples have proved to be elusive. Notice that for a smooth compact manifold M, a discrete subgroup $G \leqslant \mathrm{Hom}(M)$ is necessarily countable, since $\mathrm{Hom}(M)$ is second countable. So on smooth compact manifolds one has the following implications:



Notice that there is a simple partial result: Every topological group acting continuously, faithfully and chaotically on a Hausdorff space is totally pathwise disconnected. To see this, notice that if $G \subseteq \mathrm{Hom}(M)$ acts chaotically, then the only continuous paths in G are the constant paths. Indeed, if γ_t is a path in G and if x has finite orbit under G, then as $\gamma_t(x)$ is a continuous path in M and as $\gamma_t(x)$ belongs to the (finite) orbit of x, so $\gamma_t(x)$ is independent of t. (We remark in passing that it is easy to see that every manifold admits a non-discrete totally pathwise disconnected group of homeomorphisms.)

REFERENCES

[AdPa] ADLER, R. and R. PALAIS. Homeomorphic conjugacy of automorphisms on the torus. *Proc. Amer. Math. Soc. 16* (1965), 1222-1225.

[BBCDS] BANKS, J., J. BROOKS, G. CAIRNS, G. DAVIS and P. STACEY. On Devaney's definition of chaos. *Amer. Math. Monthly 99* (1992), 332-334.

[BFK] BRIN, M.I., J. FELDMAN and A. KATOK. Bernoulli diffeomorphisms and group extensions of dynamical systems with non-zero characteristic exponents. *Ann. of Math. 113* (1981), 159-179.

[BR] BANCHOFF, T. and M. ROSEN. Periodic points of Anosov diffeomorphisms. *Proc. Symposia in Pure Math., Vol. 14* (1970), 17-21.

[D1] DEVANEY, R. *Linked twist mappings are almost Anosov.* Lecture Notes in Math., Vol. 819 (1980), 112-145.

[D2] —— *An Introduction to Chaotic Dynamical Systems.* Addison-Wesley, 1989.

[E] EBERLEIN, P. *Structure of manifolds of nonpositive curvature.* Lecture Notes in Math., Vol. 1156 (1985), 86-153.

[FLP] FAITHI, A., F. LAUDEBACH and V. POÉNARU. Travaux de Thurston sur les surfaces. *Astérisque 66-67* (1979).

[G] GROSSMAN, E. On the residual finiteness of certain mapping class groups. *J. London Math. Soc. 9* (1973), 160-164.

[GW] GLASNER, E. and B. WEISS. Sensitive dependence on initial conditions. *Nonlinearity 6* (1993), 1067-1075.

[H] HALL, M. *The Theory of Groups.* Macmillan, New York, 1959.

[KaSp] KATOK, A. and R. SPATZIER. Differential rigidity of hyperbolic abelian actions. (Preprint.)

[KiSi] KIRBY, R. and L. SIEBENMANN. *Foundational Essays on Topological Manifolds, Smoothings, and Triangulations.* Princeton University Press, Princeton, 1977.

[LS] LYNDON, R. and P. SCHUPP. *Combinatorial Group Theory.* Springer-Verlag, Berlin, 1977.

[MKS] MAGNUS, W., A. KARRASS and D. SOLITAR. *Combinatorial Group Theory.* Wiley, New York, 1966.

[Mañ] MAÑÉ, R. *Ergodic Theory and Differentiable Dynamics.* Springer-Verlag, Berlin, 1987.

[Mann] MANNING, A. There are no new Anosov diffeomorphisms on tori. *Amer. J. Math. 96* (1974), 422-429.

[Se] SERRE, J.-P. Arithmetic groups. *Homological Group Theory* (C.T.C. Wall, ed.), pages 105-136, Cambridge University Press, Cambridge, 1979.

[Sh] SHUB, M. *Global Stability of Dynamical Systems.* Springer-Verlag, New York, 1987.

[Si] SILVERMAN, S. On maps with dense orbits and the definition of chaos. *Rocky Mountain J. Math. 22* (1992), 353-375.

[Sm] SMALE, S. Differentiable dynamical systems. *Bull. Amer. Math. Soc. 73* (1967), 747-817.

[Wa] WALTERS, P. *An Introduction to Ergodic Theory.* Springer-Verlag, New York, 1982.

[We] WEINSTEIN, M. *Examples of Groups.* Polygonal Publishing House, Passaic, 1977.

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