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Artikel: CONCERNING A REAL-VALUED CONTINUOUS FUNCTION ON THE INTERVAL WITH GRAPH OF HAUSDORFF DIMENSION 2

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Now consider the restrictions of $\tilde{f}(x)$ to all $(p+2)$-generation bands in D_L and D_R, and use the translation properties for G and its derivative g. Then by applying Lemma 2 with $\| g \| = 2^{-2^{p+2}}$, $d > 2^{-2^{p+1}-(p+2)}$, $m(I) = 2^{-n}$ we obtain

$$\mu(D_L \cap Q) + \mu(D_R \cap Q) \leqslant \left(1 + \operatorname{int} \frac{2^{-n}}{2^{-2^{p+1}-(p+2)}}\right) \cdot 2^{-2^{p+2}}. \tag{23}$$

The number of bands from the $(p+1)$ generation contained in D_0 are $2^{-n}/2^{-2^{p+1}}$, and, since $2^p < n$ by (18), we have, for $\alpha < 2$,

$$\begin{aligned} \mu(Q) = \mu(B_0 \cap Q) &\leqslant \frac{2^{-n}}{2^{-2^{p+1}}} \cdot \left(1 + \operatorname{int} \frac{2^{-n}}{2^{-2^{p+1}-(p+2)}}\right) \cdot 2^{-2^{p+2}} \\ &\leqslant 2^{-n} \cdot 2^{-2^{p+1}} + 2^{-2n+p+2} \leqslant (2^{-n})^2 \cdot (1 + 2^{p+2}) \\ &\leqslant (2^{-n})^2 (1 + 4n) \leqslant (2^{-n})^{\alpha} = | Q |^{\alpha} \end{aligned} \tag{24}$$

if $1 + 4n \leqslant 2^{n(2-\alpha)}$.

The Mass Distribution Principle now gives (17) and the proof is complete.

Remark. The nowhere-differentiability of the constructed function f is omitted in the statement of the Theorem. However this property can be established by minor changes to the proof in [RHA] or the proof of Theorem 2-9 in [D-W]. The continuity of $f(x)$ follows from uniform convergence of the series (4).

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