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Artikel: CONCERNING A REAL-VALUED CONTINUOUS FUNCTION ON THE INTERVAL WITH GRAPH OF HAUSDORFF DIMENSION 2

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Kapitel: 1. A Lemma about mass distribution

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constructed by closed dyadic cubes. The graph of a real valued function $f \in C[0, 1]$ is denoted by graph (f). By a dyadic cube we mean a cube which is the Cartesian product of dyadic intervals. If Q is an arbitrary dyadic closed cube, then the band of type $\{(x, y): (x, z) \in Q \text{ for some } z \in \mathbf{R}\}$ is called a dyadic band. In our construction the dyadic bands of width 2^{-2^p} play a special role. They are called bands of generation $p, p = 0, 1, 2, \ldots$.

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1. A Lemma about mass distribution

By a mass distribution on a subset A of $\mathbf{R}^2$ we mean a measure μ on A such that $0 < \mu(A) < \infty$.

LEMMA 1. *Let f be a real valued measurable function defined on $[0, 1]$. Then there is a mass distribution μ on $F := \text{graph}(f)$ such that*

1) *for any two subintervals I and I' of $[0, 1]$, with $m(I) = m(I')$,*

$$\mu(I \times \mathbf{R}) = \mu(I' \times \mathbf{R})$$

and

2) *if for two Borel sets B_1 and B_2 in $[0, 1] \times \mathbf{R}$ there exists $(x_0, y_0) \in \mathbf{R}^2$ such that*

$$B_1 \cap F + (x_0, y_0) = B_2 \cap F$$

then

$$\mu(B_1) = \mu(B_2) .$$

Proof. Let B be an arbitrary Borel set in $\mathbf{R}^2$. Define

$$\mu(B) = m(\tilde{f}^{-1}(B)) . \tag{5}$$

Then it is obvious that μ is a mass distribution on graph(f) and 1) and 2) follow from the translation invariance of the Lebesgue measure.

2. A Lemma about mass distribution and successive translations

LEMMA 2. *Let $g(y) \geqslant 0$ and $g(y) \in L^1(\mathbf{R})$. If I is a finite interval and d is a positive real number then*

$$\int_I \sum_{n=-\infty}^{\infty} g(y - nd)\,dy < \left(1 + \text{int}\,\frac{m(I)}{d}\right) \cdot \|g\| . \tag{6}$$