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$$\begin{array}{ccc}
 H(G) & \xrightarrow{\psi^*} & [H(G/T) \otimes H(T)]^W . \\
 p_k^* \downarrow & & \downarrow 1 \otimes p_k^* \\
 H(G) & \xrightarrow{\psi^*} & [H(G/T) \otimes H(T)]^W .
 \end{array}$$

Since  $p_k^*$  acts by  $k^q$  on  $H^q(T)$ , (6.1) implies that  $H^n(G)_q \simeq [H^{n-q}(G/T) \otimes H^q(T)]^W$ , and (6.3) gives the dimension of the latter space.

(6.6) This last interpretation of the bigrading shows that it is natural in the following sense. Suppose  $f: K \rightarrow G$  is a homomorphism between two compact connected Lie groups. Since  $f$  commutes with the power maps  $P_k$  on  $G$  and  $K$ , the cohomology map  $f^*$  sends  $H^n(G)_q$  to  $H^n(K)_q$ . Suppose for example that  $K$  is a closed connected subgroup of  $G$  and  $f$  is the inclusion map. Choose, as we may, a maximal torus  $T$  of  $G$  such that  $S := T \cap K$  is a maximal torus of  $K$ . The restriction map  $H(G) \rightarrow H(K)$  becomes, via (6.1), the map  $[H(G/T) \otimes H(T)]^W \rightarrow [H(K/S) \otimes H(S)]^{W_K}$  induced by restriction on each factor, where  $W_K$  is the Weyl group of  $S$  in  $K$ .

(6.7) We close with the homology interpretation of (6.1), which says the homology map  $\psi_*$  induced by  $\psi$  is surjective. It follows that the homology of  $G$  is spanned by the cycles  $[\psi(\bar{X}_w, T_I)] = \{gtg^{-1}: gT \in \bar{X}_w, t \in T_I\}$ . Here  $w \in W$ ,  $X_w$  is the Schubert cell (see (5.2)) and  $T_I = \prod_{i \in I} T_i$ , where  $T = T_1 \times \cdots \times T_l$ , with each  $T_i \simeq S^1$ . Using the results in [BGG], one can explicitly write down the action of  $W$  on  $H_*(G/T)$  in terms of the Schubert cell basis, and this leads, in principle, to the linear relations in  $H_*(G)$  satisfied by the cycles  $[\psi(\bar{X}_w, T_I)]$ .

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