Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 40 (1994)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: ENUMERATIVE COMBINATORICS AND CODING THEORY
Autor: Eliahou, Shalom
Kurzfassung
DOI: https://doi.org/10.5169/seals-61109

L'Enseignement Mathématique, t. 40 (1994), p. 171-185

ENUMERATIVE COMBINATORICS AND CODING THEORY

by Shalom ELIAHOU [1])

ABSTRACT. Let f be a polynomial in n variables with non-negative integral coefficients. The enumeration of the values assumed by f on $\{1, -1\}^n$ is shown to be equivalent to the enumeration of the weights in some associated binary linear code L_f. We use this correspondence, together with the MacWilliams identity, to enumerate (1) Hadamard matrices of some fixed order, and (2) the proper 4-colorings of a graph, in terms of the weight distribution of suitable binary codes. Similar formulas could be obtained for other combinatorial objects.[2])

Let f be a polynomial in n variables with non-negative integral coefficients. We will address here the following

PROBLEM. Is there a point $p \in \{1, -1\}^n$ such that $f(p) = 0$? How many such binary zeros does f admit? More generally, what can be said about the *value enumerator* of f, which we define as

$$V_f(T) = \sum_{p \in \{\pm 1\}^n} T^{f(p)} \in \mathbf{N}[T, T^{-1}] \ ?$$

(Note that the coefficient of T^v in $V_f(T)$ is the number of binary points p such that $f(p) = v$, for $v \in \mathbf{Z}$.) Many classical combinatorial problems can be expressed in the above terms, for a suitable polynomial f.

In order to discuss these problems, we associate with f a binary linear code L_f, in such a way that the weight enumerator of L_f and the value enumerator of f faithfully reflect each other. We then invoke the MacWilliams identity from coding theory, to obtain formulas for the number of binary zeros

[1]) The author gratefully acknowledges support from the Fonds National Suisse de la Recherche Scientifique during part of the preparation of this paper.

[2]) MR classification primary 05A15, secondary 94B05, 05B20, 05C15.