

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 40 (1994)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: AN EXPOSITION OF POINCARÉ'S POLYHEDRON THEOREM
Autor: Epstein, David B.A. / Petronio, Carlo
Bibliographie
DOI: <https://doi.org/10.5169/seals-61108>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Siehe Rechtliche Hinweise.

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. Voir Informations légales.

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. See Legal notice.

Download PDF: 20.05.2025

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

A pyramidal cell in \mathbf{R}^{n-1} corresponds to a convex cell in \mathbf{H}^n together with an ideal point p in its boundary, such that any two faces with closures containing p meet inside \mathbf{H}^n . A non-pyramidal cell corresponds to a convex cell in \mathbf{H}^n and an ideal point p contained in the closures of two non-intersecting faces of the convex cell. The hypothesis needed in order to apply Theorem 10.1, that there are only a finite number of orbits of non-pyramidal cells, comes from the fact that there are only a finite number of pairs of faces and therefore only a finite number of pairs of non-intersecting faces which meet at infinity.

It follows that the inverse image in X of any point of \bar{Q} is finite. Moreover the number of points in the inverse image is bounded by a fixed integer N . Two points $x, y \in X$ are mapped to the same point of \bar{Q} if and only if there is a sequence (x_0, \dots, x_n) such that $x = x_0$, $y = x_n$ and $x_{i+1} = A(F_i)(x_i)$, where $x_i \in F_i$ and $x_{i+1} \in R(F_i)$. (Here (R, A) is the glueing data.) We may take $n \leq N$. It follows easily from compactness and the finiteness of the situation that the map $X \rightarrow \bar{Q}$ is closed. Therefore \bar{Q} is hausdorff. \square

REFERENCES

- [Ale54] ALEKSANDROV, A.D. Filling space by polyhedra. *Vestnik. Leningrad Univ. Math.* 2 (1954), 33-34.
- [Apa86] APANASOV, B.N. Filling a space by polyhedra and deformation of incomplete hyperbolic structures. *Siberian Math. J.* 27 (1986), 473-485. (English translation.)
- [Bea83] BEARDON, A.F. *The Geometry of Discrete Groups*. Springer-Verlag, 1983.
- [Bow93] BOWDITCH, B. Geometrical finiteness for hyperbolic groups. *J. Funct. Anal.* 113 (1993), 245-317.
- [BP92] BENEDETTI, R. and C. PETRONIO. *Lectures on hyperbolic geometry*. Springer-Verlag, 1992.
- [BSS89] BLUM, L., M. SHUB and S. SMALE. On a theory of computation and complexity over the real numbers: NP-completeness, recursive functions and universal machines. *Bull. Amer. Math. Soc.* 21 (1989), 1-46.
- [dlH91] DE LA HARPE, P. An invitation to Coxeter groups. In: *Group theory from a geometrical viewpoint*, Ghys-Haefliger-Verjovsky editors, World Scientific Publishers, Singapore, 1991.
- [dR71] DE RHAM, G. Sur les polygones génératrices de groupes fuchsiens. *Enseign. Math.* 17 (1971), 49-61.
- [Mas71] MASKIT, B. On Poincaré's theorem for fundamental polygons. *Adv. Math.* 7 (1971), 219-230.
- [Mas88] —— *Kleinian Groups*. Springer-Verlag, 1988.
- [Mor78] MOROKUMA, T. A characterization of fundamental domains of discontinuous groups acting on real hyperbolic spaces. *J. Fac. Sci. Univ. Tokyo Section 1A Math.* 25 (1978), 157-183.

- [Poi82] POINCARÉ, H. Théorie des groupes fuchsiens. *Acta Math.* 1 (1882), 1-62.
- [Poi83] —— Mémoire sur les groupes Kleinéens. *Acta Math.* 3 (1883), 49-92.
- [Poi52] —— *Collected Works*. Gauthier-Villars, 1952.
- [Ril83] RILEY, R. Applications of a computer implementation of Poincaré's theorem on fundamental polyhedra. *Math. Comp.* (1983), 607-632.
- [Sei75] SEIFERT, H. Komplexe mit Seitenzuordnung. *Göttinger Nachrichten* (1975), 49-80.
- [Thu] THURSTON, W.P. The geometry and topology of three-dimensional manifolds. (In preparation.)
- [Thu80] —— *The Geometry and Topology of Three-Manifolds*. Princeton University Mathematics Department, 1980. (Thurston's original notes.)

(Reçu le 10 novembre 1993)

David B.A. Epstein

University of Warwick (Coventry, England)
and Geometry Center (Minneapolis, USA)

Carlo Petronio

Università di Pisa (Italy)