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A pyramidal cell in  $\mathbf{R}^{n-1}$  corresponds to a convex cell in  $\mathbf{H}^n$  together with an ideal point  $p$  in its boundary, such that any two faces with closures containing  $p$  meet inside  $\mathbf{H}^n$ . A non-pyramidal cell corresponds to a convex cell in  $\mathbf{H}^n$  and an ideal point  $p$  contained in the closures of two non-intersecting faces of the convex cell. The hypothesis needed in order to apply Theorem 10.1, that there are only a finite number of orbits of non-pyramidal cells, comes from the fact that there are only a finite number of pairs of faces and therefore only a finite number of pairs of non-intersecting faces which meet at infinity.

It follows that the inverse image in  $X$  of any point of  $\bar{Q}$  is finite. Moreover the number of points in the inverse image is bounded by a fixed integer  $N$ . Two points  $x, y \in X$  are mapped to the same point of  $\bar{Q}$  if and only if there is a sequence  $(x_0, \dots, x_n)$  such that  $x = x_0$ ,  $y = x_n$  and  $x_{i+1} = A(F_i)(x_i)$ , where  $x_i \in F_i$  and  $x_{i+1} \in R(F_i)$ . (Here  $(R, A)$  is the glueing data.) We may take  $n \leq N$ . It follows easily from compactness and the finiteness of the situation that the map  $X \rightarrow \bar{Q}$  is closed. Therefore  $\bar{Q}$  is hausdorff.  $\square$

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