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A pyramidal cell in $\mathbf{R}^{n-1}$ corresponds to a convex cell in $\mathbf{H}^n$ together with an ideal point p in its boundary, such that any two faces with closures containing p meet inside $\mathbf{H}^n$. A non-pyramidal cell corresponds to a convex cell in $\mathbf{H}^n$ and an ideal point p contained in the closures of two non-intersecting faces of the convex cell. The hypothesis needed in order to apply Theorem 10.1, that there are only a finite number of orbits of non-pyramidal cells, comes from the fact that there are only a finite number of pairs of faces and therefore only a finite number of pairs of non-intersecting faces which meet at infinity.

It follows that the inverse image in X of any point of $\bar{Q}$ is finite. Moreover the number of points in the inverse image is bounded by a fixed integer N. Two points $x, y \in X$ are mapped to the same point of $\bar{Q}$ if and only if there is a sequence $(x_0, \ldots, x_n)$ such that $x = x_0$, $y = x_n$ and $x_{i+1} = A(F_i)(x_i)$, where $x_i \in F_i$ and $x_{i+1} \in R(F_i)$. (Here (R, A) is the glueing data.) We may take $n \leqslant N$. It follows easily from compactness and the finiteness of the situation that the map $X \to \bar{Q}$ is closed. Therefore $\bar{Q}$ is hausdorff. □

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