

Zeitschrift: L'Enseignement Mathématique
Herausgeber: Commission Internationale de l'Enseignement Mathématique
Band: 40 (1994)
Heft: 1-2: L'ENSEIGNEMENT MATHÉMATIQUE

Artikel: NOTE ON TABLE I OF "BARKER SEQUENCES AND DIFFERENCE SETS"
Autor: Broughton, Wayne J.
DOI: <https://doi.org/10.5169/seals-61106>

Nutzungsbedingungen

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

Conditions d'utilisation

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

Terms of use

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

Download PDF: 27.04.2026

ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>

A NOTE ON TABLE I
OF "BARKER SEQUENCES AND DIFFERENCE SETS"

by Wayne J. BROUGHTON

In Table I of [EK], S. Eliahou and M. Kervaire show the non-existence of cyclic difference sets with parameters $(2t(t+1)+1, t^2, t(t-1)/2)$, for $3 \leq t \leq 100$, $t \neq 50$, leaving the case $t = 50$ undecided. The purpose of this note is to fill this gap and to generalize the table to non-cyclic difference sets. See any of [EK], [L], or [J] for definitions and notation.

To handle the case $t = 50$ we make use of a multiplier theorem due to McFarland (see [L], Theorem 5.24, p. 218, or [J], Theorem 4.7, p. 254). It refers to a function $M(z)$ which has $M(1) = 1$, and (for $z \geq 5$) is defined recursively to be the product of the distinct prime factors of the numbers

$$z, M\left(\frac{z^2}{p^{2e}}\right), p-1, p^2-1, \dots, p^{u(z)}-1,$$

where p is any prime dividing z with $p^e \parallel z$ and where $u(z) = (z^2 - z)/2$. (Note that the "definition" of M depends on the choice of p made for each z .)

PROPOSITION. *If D is an abelian (v, k, λ) -difference set in G , and m is a divisor of $n := k - \lambda$ such that $M(n/m)$ and v are co-prime, and if d is an integer co-prime with v such that for every prime $p \mid m$ there exists $f \geq 0$ with $p^f \equiv d \pmod{\exp(G)}$, then d is a numerical multiplier of D .*

Now when $t = 50$ we have $v = 5101$, a prime, (so $G = \mathbf{Z}_{5101}$), and $n = 1275 = 3 \cdot 5^2 \cdot 17$. Let $m = 3 \cdot 17$. So $n/m = 5^2$, and $M(5^2)$ has as factors the prime factors of

$$5^2, M(1), 5-1, 5^2-1, \dots, 5^{300}-1,$$

since $u(25) = 300$. But the multiplicative order of 5 modulo 5101 is 425, so $M(25)$ is not divisible by $v = 5101$. Moreover,

$$3^{1088} \equiv 17^1 \pmod{5101},$$

so by the proposition $d = 17$ is a multiplier of any $(5101, 2500, 1225)$ -difference set.

But the non-trivial orbits of multiplication by 17 in \mathbf{Z}_{5101} are all of size 75, so it is impossible for a union of orbits to have size 2500 and hence no such difference set exists.

The primary non-existence theorem used in Table I of [EK] to eliminate difference sets is what they call the Semi-Primitivity Theorem (see Theorem 4.5 of [L] or Theorem 7.1 of [J]). Since this theorem actually applies to abelian difference sets (not just cyclic ones), it can also be used to eliminate almost all of the abelian difference sets in the range $3 \leq t \leq 100$. The only (non-cyclic) abelian case to which the theorem does not apply is $t = 49$, where the parameters are $(4901, 2401, 1176)$ and $n = 1225 = 35^2$. This is easily eliminated by Theorem 4.18 of [L]. Since $4901 = 13^2 \cdot 29$, we can (using Lander's notation) take a subgroup H in G of order $h = 29$, and let $m = 35$. So $m^2 \mid n$, and m is semi-primitive mod $|G/H| = 169$ since $5^{26} \equiv 7^{78} \equiv -1 \pmod{169}$; but by the theorem this implies $h \geq m$ (note the misprint in [L]), which is a contradiction.

Next, the only values of $t \in \{3, \dots, 100\}$ for which there exists a non-abelian group of order $v = 2t(t+1) + 1$ are $t = 26, 28, 36, 41, 48, 51, 52, 66, 73, 76, 86, 88, 96$, and 98 . In every one of these cases we can apply Theorem 4.4 of [L] (Theorem 7.6 in [J]), using the semi-primitivity relations already listed in Table I of [EK].

So we conclude that there do not exist *any* $(2t(t+1) + 1, t^2, t(t-1)/2)$ -difference sets for $3 \leq t \leq 100$.

We now point out a few misprints in Table I:

- (i) At $t = 12$, v should be "313" (a prime), not " $3 \cdot 13$ ".
- (ii) At $t = 17$, the semi-primitivity relation should read " $3^{51} \equiv -1 \pmod{613}$ ".
- (iii) At $t = 28$, the factorization for n should read " $2 \cdot 7 \cdot 29$ ".
- (iv) At $t = 61$, v should be " $5 \cdot 17 \cdot 89$ ".

S. Eliahou and M. Kervaire have also pointed out that on page 375 the polynomial $\theta_0(y)$ should read

$$y^3 + y^6 + y^7 + y^9 + y^{11} + y^{12} + y^{13} + y^{14}.$$

Finally, they also requested mention of the fact that at the time of writing [EK], they were not aware of the paper [C], which contains the complete classification of $(255, 127, 63)$ cyclic difference sets and should have been included in their bibliography.

REFERENCES

- [C] CHENG, U. Exhaustive Construction of $(255, 127, 63)$ -Cyclic Difference Sets. *J. Comb. Theory., Ser. A* 35 (1983), No. 2, 115-125.
- [EK] ELIAHOU, S. and M. KERVAIRE. Barker Sequences and Difference Sets. *L'Ens. Math.* 38 (1992), 345-382.
- [J] JUNGnickel, D. Difference Sets. In *Contemporary Design Theory*, J. H. Dinitz and D. R. Stinson, Ed., Wiley-Interscience (1992), 241-324.
- [L] LANDER, E. S. *Symmetric Designs: An Algebraic Approach*. London Math. Soc. Lecture Note Series 74, Cambridge University Press (1983).

(Reçu le 21 décembre 1993)

Wayne J. Broughton

Department of Mathematics
California Institute of Technology
Sloan 253-37
Pasadena, CA 91125
U.S.A.

Vide-leer-empty