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Artikel: UNIMODULAR LATTICES WITH A COMPLETE ROOT SYSTEM

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(4) $8A_4$

Let $e_1 = (1,0,0,0)$, $e_2 = (0,1,0,0)$, $e_3 = (0,0,1,0)$, $e_4 = (0,0,0,1)$. Any metabolizer must have a basis of the form $\{e_i + v_i, i = 1,2,3,4\}$ for some vectors $v_i \in \mathbf{F}_5^4$ of weight 3 or 4.

Hence, we may assume that the first basis vector is either $s_1 = e_1 + (1,1,1,1)$ or $t_1 = e_1 + (0,1,2,2)$.

If we start with s_1, there are essentially only 2 ways of completing s_1 to an admissible metabolizer with 3 vectors forming with s_1 the rows of the matrix S exhibited in the table and the matrix S':

$$S' = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 2 & 2 \\ 0 & 0 & 1 & 0 & 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 & 2 & 2 & 0 \end{pmatrix}.$$

If we start with t_1 there is essentially only one way to complete to a metabolizer:

$$S'' = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 2 & 2 \\ 0 & 1 & 0 & 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 0 & 3 & 3 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 & 3 & 1 & 0 \end{pmatrix}.$$

All these metabolizers are equivalent. The permutation ρ defined by

$$\rho(x_0, \ldots, x_7) = (x_4, x_1, x_2, -x_3, x_7, x_5, x_6, x_0)$$

sends S' to S and σ defined by

$$\sigma(x_0, \ldots, x_7) = (x_5, x_1, x_4, x_0, x_7, x_2, x_3, x_6)$$

sends S'' to S.

Thus the lattice described by the filling set S is the only one with the root system $\mathbf{8A_4}$.

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