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6. Open Problems

There are many open questions and unproven conjectures about the Prouhet-Tarry-Escott problem. We conclude by listing a few.

1. Find an ideal solution for any size higher than 10 or find some degree for which an ideal solution does not exist. (Even a heuristic argument would be of interest.)
2. Find another class of solutions of size 9 or 10.
3. Prove $N(k) \leqslant o(k^2)$.
4. Prove $M(k) \leqslant O(k^2)$.
5. Show that there is no 7 factor (degree 6) pure product of norm 14.
6. Find a non-trivial lower bound for $A(k)$. Almost equivalently prove

$$\min_{n_1, \ldots, n_k} \left\| \prod_{i=1}^{k} (1 - x^{n_k}) \right\|_1 > 2k$$

   for some k. (Problem 5 is the $k = 7$ case of this.)
7. Find a true algorithm, even an impractical one, that determines if there is an ideal solution of size 11.
8. Find a true algorithm, even an impractical one, that determines if there is a degree 6 ($k = 7$) pure product of norm 14.
9. Solve the ideal problem mod p^n for all primes p smaller than the size of the solution and all n.

The big prize is to find ideal solutions of all degrees, if indeed they exist. Question 1 above is, of course, the first step. No progress on questions 3 and 4 has been made for many years. Questions 5, 6, and 8 all relate to the Erdős-Szekeres Problem. The issue in Questions 7 and 8 is that it is not known how to bound solutions so as to make the problems finite. Question 9 is raised in [17] and would show that no local obstructions exist to solutions.

REFERENCES

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