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Kapitel: 2. Aspects of geometry

Perhaps one of the main reasons for this situation is that geometry has so many aspects, and as a consequence there has not yet been found — and perhaps there does not exist at all — a simple, clean, linear "hierarchical" path from the first beginnings to the more advanced achievements of geometry. Unlike what happens in arithmetic and algebra, even basic concepts in geometry, such as the notions of angle and distance, have to be reconsidered at different stages from different viewpoints.

Another problematic point concerns the role of proofs in geometry: relations between intuition, inductive and deductive proofs, age of students at which proofs can be introduced, and different levels of rigour and abstraction.

Thus the teaching of geometry is not at all an easy task. But instead of trying to face and overcome the obstacles arising in the teaching of geometry, actual school-practice in many countries has simply bypassed these obstacles, cutting out the more demanding parts, often without any replacement. For instance, three-dimensional geometry has almost disappeared or has been confined to a marginal role in the curricula in most countries.

Starting from this analysis, and specifically considering the gap between the increasing importance of geometry for its own sake, as well as in research and in society, and the decline of its role in school curricula, ICMI feels that there is an urgent need for an international study, whose main aims are:

— To discuss the goals of the teaching of geometry at different school levels and according to different cultural traditions and environments.

— To identify important challenges and emerging trends for the future and to analyze their potential didactical impact.

— To exploit and implement new teaching methods.

2. Aspects of geometry

The outstanding historical importance of geometry in the past, in particular as a prototype of an axiomatic theory, is so universally acknowledged that it deserves no further comment. Moreover, in the last century and specifically during the last decades, as Jean Dieudonné asserted at ICME 4 (Berkeley, 1980), Geometry "bursting out of its traditional narrow confines [...] has revealed its hidden powers and its extraordinary versatility and adaptability, thus becoming one of the most universal and useful tools in all parts of mathematics" (J. Dieudonné: The Universal Domination of Geometry, ZDM 13 (1), p. 5-7 (1981)).

Actually, geometry includes so many different aspects, that it is hopeless (and maybe even useless) to write out a complete list of them. Here we mention only those aspects, which in our opinion are particularly relevant in view of their didactical implications:

— Geometry, as *the science of space.* From its roots as a tool for describing and measuring figures, geometry has grown into a theory of ideas and methods by which we can construct and study idealized models of the physical world as well as of other real world phenomena. According to different points of view, we get euclidean, affine, descriptive, projective geometry, but also topology or non-euclidean and combinatorial geometries.

— Geometry as *a method for visual representations of concepts and processes* from other areas in mathematics and in other sciences; e.g. graphs and graph theory, diagrams of various kinds, histograms.

— Geometry as *a meeting point* between mathematics as a theory and mathematics as a model resource.

— Geometry as *a way of thinkings and understanding* and, at a higher level, as *a formal theory.*

— Geometry as *a paradigmatic example for teaching deductive reasoning.*

— Geometry as *a tool in applications,* both traditional and innovative. The latter ones include e.g. computer graphics, image processing and image manipulation, pattern recognition, robotics, operations research.

Another distinction should be made with respect to several different approaches according to which one may deal with geometry. Roughly speaking, possible approaches are:

- manipulative;
- intuitive;
- deductive;
- analytic.

Also one may distinguish between a geometry which stresses "static" properties of geometric objects and a geometry where objects are considered in a "dynamic" setting, as they change under the effect of different types of space transformations.