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THE PROUHET-TARRY-ESCOTT PROBLEM REVISITED

by Peter BORWEIN and Colin INGALLS

ABSTRACT. The old problem of Prouhet, Tarry, Escott and others asks one to find two distinct sets of integers $\{\alpha_1, \dots, \alpha_n\}$, and $\{\beta_1, \dots, \beta_n\}$ with

$$\alpha_1^m + \cdots + \alpha_n^m = \beta_1^m + \cdots + \beta_n^m$$

for $m = 1, \dots, k$ (with the most interesting case being $k = n - 1$). We review some elementary properties of solutions and examine the fine structure of 'ideal' and 'symmetric ideal' solutions. The relationship of this problem to the 'easier' Waring problem and a problem of Erdős and Szekeres of minimizing the norm of a product of cyclotomic polynomials on the unit disk is then discussed. We present some new bounds for this problem and for the Prouhet-Tarry-Escott problem of small size. We also present an algorithm for calculating symmetric ideal p-adic solutions of the the Prouhet-Tarry-Escott problem.

1. INTRODUCTION

A classic problem in Diophantine Analysis that occurs in many guises is the Prouhet-Tarry-Escott problem. This is the problem of finding two distinct sets of integers $\{\alpha_1, \dots, \alpha_n\}$, $\{\beta_1, \dots, \beta_n\}$ such that

$$\begin{aligned}
\alpha_1 + \cdots + \alpha_n &= \beta_1 + \cdots + \beta_n \\
\alpha_1^2 + \cdots + \alpha_n^2 &= \beta_1^2 + \cdots + \beta_n^2 \\
&\vdots \\
\alpha_1^k + \cdots + \alpha_n^k &= \beta_1^k + \cdots + \beta_n^k .
\end{aligned}$$

Classification Numbers: 11-04, 11D41. Key Words: Diophantine Equations, Tarry, Escott, Prouhet, Waring Problem.