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## AN ERGODIC ADDING MACHINE ON THE CANTOR SET

by Peter COLLAS and David KLEIN

ABSTRACT. We calculate all ergodic measures for a specific function  $F$  on the unit interval. The supports of these measures consist of periodic orbits of period  $2^n$  and the classical ternary Cantor set. On the Cantor set,  $F$  is topologically conjugate to an “adding machine” in base 2. We show that  $F$  is representative of the class of functions with zero topological entropy on the unit interval, already analyzed in the literature, and its behavior is therefore typical of that class.

### I. INTRODUCTION

The dynamical behavior of the quadratic function  $f_c(x) = x^2 - c$  has been extensively studied as the parameter  $c$  is varied. For example,  $c_0 = 1.401155189\dots$  is the smallest value of  $c$  for which  $f_c(x)$  has infinitely many distinct periodic orbits [1-3]. As  $c$  approaches this number through smaller values, the dynamical system,  $x \rightarrow f_c(x)$ , progresses through the famous period doubling route to chaos. When  $c = c_0$ , the dynamical behavior of  $f(x) \equiv f_c(x)$  includes the following properties:

1. There is a Cantor set  $K$  which is an attractor and  $f : K \rightarrow K$
2. All periodic points of  $f$  have period  $2^n$  for some  $n$ .
3. There are periodic points which are arbitrarily close to  $K$ .
4. With the restriction of  $f(x)$  to an appropriate interval  $I$  such that  $f(I) \subset I$ , there are just two possibilities for the orbit of a point  $x_0 \in I$ : either  $f^k(x_0)$  is in a periodic orbit for some  $k$ , or  $f^k(x_0)$  converges to  $K$  as  $k$  increases.
5. The restriction of  $f$  to  $K$  is topologically equivalent to a function on 2-adic integers which adds 1 to its argument (this “adding machine” will be described in detail below).