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Traditionally, p -adic analysis has been done in \mathbf{C}_p . But every power series $F(X) = \sum_{n=0}^{\infty} a_n X^n$ with $a_n \in \mathbf{C}_p$ can be defined on L , and the radius of convergence is the same in L as in \mathbf{C}_p , because in either field the series converges iff the valuation of its terms approach $+\infty$. (Remember that L is complete.) As an example, we state the following proposition.

PROPOSITION 13. *There exists a unique function $\log_p: L^* \rightarrow L$ such that*

- (1) $\log_p x = \sum_{n=1}^{\infty} (-1)^{n+1} (x-1)^n / n$, for $v(x-1) > 0$.
- (2) $\log_p xy = \log_p x + \log_p y$, for all $x, y \in L^*$.
- (3) $\log_p p = 0$.

Proof. The proof for L is exactly the same as the proof for \mathbf{C}_p . See pp. 87-88 in [7]. \square

Although we can extend any power series defined on \mathbf{C}_p to L , it seems that p -adic analysis rarely (if ever) would need to use properties of L not true of \mathbf{C}_p . All that seems important is that the field is a complete algebraically closed immediate extension of $\bar{\mathbf{Q}}_p$. It would be interesting to investigate whether anything can be gained by doing p -adic analysis in L instead of in \mathbf{C}_p .

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