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Autor: Poonen, Bjorn
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Traditionally, p-adic analysis has been done in $\mathbf{C}_p$. But every power series $F(X) = \sum_{n=0}^{\infty} a_n X^n$ with $a_n \in \mathbf{C}_p$ can be defined on L, and the radius of convergence is the same in L as in $\mathbf{C}_p$, because in either field the series converges iff the valuation of its terms approach $+\infty$. (Remember that L is complete.) As an example, we state the following proposition.

PROPOSITION 13. *There exists a unique function* $\log_p: L^* \to L$ *such that*

(1) $\log_p x = \sum_{n=1}^{\infty} (-1)^{n+1}(x-1)^n/n$, *for* $v(x-1) > 0$.

(2) $\log_p xy = \log_p x + \log_p y$, *for all* $x, y \in L^*$.

(3) $\log_p p = 0$.

Proof. The proof for L is exactly the same as the proof for $\mathbf{C}_p$. See pp. 87-88 in [7]. □

Although we can extend any power series defined on $\mathbf{C}_p$ to L, it seems that p-adic analysis rarely (if ever) would need to use properties of L not true of $\mathbf{C}_p$. All that seems important is that the field is a complete algebraically closed immediate extension of $\bar{\mathbf{Q}}_p$. It would be interesting to investigate whether anything can be gained by doing p-adic analysis in L instead of in $\mathbf{C}_p$.

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Bjorn Poonen
Department of Mathematics
University of California at Berkeley
Berkeley, CA 94720

E-mail address: poonen@math.berkeley.edu