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then we expect to find zeros in a neighborhood of each point of

$$w - w^{d+1} (f'(w))^{-1} Q(w) .$$

The set  $Q(w)$  is connected [1], and for  $w \notin \mathbf{R}$ , it seems that it contains a small disk around the origin. The set  $Q(w)$  is a continuous function of  $w$ , which accounts for the similarity of the protrusions from  $\bar{W}$  visible in Figures 5 and 6. (The protrusions in Figure 4 are different, since there the sets  $Q(w)$  are of different shape from those in Figures 5 and 6.)

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