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3.9. *Example.* Let $\mathbf{X} := \{x^3 + y^4 + z^3 = 0\} \subset \mathbf{C}^3$ and let $f \in \mathbf{m}_{\mathbf{X},0}$ be the function induced by $\tilde{f} \in \mathbf{m}_{\mathbf{C}^3,0}$, $\tilde{f} = x$. Consider the linear function l induced by $\tilde{l} = y$. Then $l \in \Omega_f$. We get that $\Delta(l, f)$ is irreducible and has the Puiseux parametrization: $l = \alpha v^3$, $\lambda = v^4$, where α is a nonzero constant, easy to determine.

Let $c \in \Delta(l, f) \cap (D_\alpha \times \{\eta\})$ and let $a \notin \Delta(l, f) \cap (D_\alpha \times \{\eta\})$ be a neighbour point of c.

The monodromy h'_a can be identified to the monodromy of the function $f_a: (\mathbf{X}_a, 0) \to (\mathbf{C}, 0)$ induced by $\tilde{f}_a = v$, where $\mathbf{X}_a := \{x = v^4, y = v^3, z = \sqrt[3]{2}\gamma v^4\}$ and γ is a 3-root of -1. We get $\zeta_{h'_a}(t) = (1-t)^{-3}$, hence $\zeta_{h_c^{\text{rel}}} = (1-t)^2$.

By using (8), the final result is $\zeta_f(t) = (1-t)^{-3}(1-t^4)^2$. We also get $\Lambda(f) = 3$.

Notice that there is another way of computing the zeta function in this example, by using the usual $\mathbf{C}^*$-action on $\mathbf{X}$, which fixes the zero set $\{\tilde{f} = 0\}$. It follows that the monodromy h_f of f is equal to the 3rd power of the monodromy h_g of the function $g: (\mathbf{C}^2, 0) \to (\mathbf{C}, 0)$, $g = y^4 + z^3$ and $\zeta_{h_g^3}(t)$ can be computed from the eigenvalues of h_g. If we change the above function $\tilde{f}$ into $\tilde{f}_1 := x + y$, then the set $\{\tilde{f}_1 = 0\}$ is no more invariant under the above-mentioned $\mathbf{C}^*$-action. The computations for the zeta-function of h_{f_1} are slightly more complicated, since we get two Puiseux pairs, with $n_{1,1} = 1$, $n_{1,2} = 3$. This time, the result is $\zeta_{f_1}(t) = (1-t)^{-1}(1-t^3)^{-1}(1-t^9)$.

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