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3.9. *Example.* Let $\mathbf{X} := \{x^3 + y^4 + z^3 = 0\} \subset \mathbf{C}^3$ and let $f \in \mathfrak{m}_{\mathbf{X},0}$ be the function induced by $\tilde{f} \in \mathfrak{m}_{\mathbf{C}^3,0}$, $\tilde{f} = x$. Consider the linear function l induced by $\tilde{l} = y$. Then $l \in \Omega_f$. We get that $\Delta(l, f)$ is irreducible and has the Puiseux parametrization: $l = \alpha v^3$, $\lambda = v^4$, where α is a nonzero constant, easy to determine.

Let $c \in \Delta(l, f) \cap (D_\alpha \times \{\eta\})$ and let $a \notin \Delta(l, f) \cap (D_\alpha \times \{\eta\})$ be a neighbour point of c .

The monodromy h'_a can be identified to the monodromy of the function $f_a: (\mathbf{X}_a, 0) \rightarrow (\mathbf{C}, 0)$ induced by $\tilde{f}_a = v$, where $\mathbf{X}_a := \{x = v^4, y = v^3, z = \sqrt[3]{2\gamma v^4}\}$ and γ is a 3-root of -1 . We get $\zeta_{h'_a}(t) = (1-t)^{-3}$, hence $\zeta_{h_c^{\text{rel}}} = (1-t)^2$.

By using (8), the final result is $\zeta_f(t) = (1-t)^{-3}(1-t^4)^2$. We also get $\Lambda(f) = 3$.

Notice that there is another way of computing the zeta function in this example, by using the usual \mathbf{C}^* -action on \mathbf{X} , which fixes the zero set $\{\tilde{f} = 0\}$. It follows that the monodromy h_f of f is equal to the 3rd power of the monodromy h_g of the function $g: (\mathbf{C}^2, 0) \rightarrow (\mathbf{C}, 0)$, $g = y^4 + z^3$ and $\zeta_{h_g^3}(t)$ can be computed from the eigenvalues of h_g . If we change the above function \tilde{f} into $\tilde{f}_1 := x + y$, then the set $\{\tilde{f}_1 = 0\}$ is no more invariant under the above-mentioned \mathbf{C}^* -action. The computations for the zeta-function of h_{f_1} are slightly more complicated, since we get two Puiseux pairs, with $n_{1,1} = 1$, $n_{1,2} = 3$. This time, the result is $\zeta_{f_1}(t) = (1-t)^{-1}(1-t^3)^{-1}(1-t^9)$.

REFERENCES

- [BK] BRIESKORN, E und H. KNÖRER. *Ebene algebraische Kurven*. Birkhäuser Verlag, 1981.
- [A'C-1] A'CAMPO, N. Le nombre de Lefschetz d'une monodromie. *Indag. Math.* 35 (1973), 113-118.
- [A'C-2] ——— La fonction zêta d'une monodromie. *Comment. Math. Helvetici* 50 (1975), 233-248.
- [EN] EISENBUD, D. and W. NEUMANN. *Three-dimensional link theory and invariants of plane curve singularities*. Annals of Math. Studies 110 (1985), Princeton Univ. Press.
- [Lê-1] LÊ, D.T. The geometry of the monodromy theorem. C.P. Ramanujam — a tribute, Tata Institute, Springer-Verlag.
- [Lê-2] ——— Some remarks on the relative monodromy. *Real and Complex Singularities* Oslo 1976, Sijhoff en Nordhoff, Alphen a.d. Rijn 1977, 397-403.
- [Lê-3] ——— La théorème de monodromie singulière. *C.R. Acad. Sci. Paris t. 288, III* (1979), 985-988.

- [Lê-4] LÊ, D.T. Calcul du nombre de cycles évanouissants d'une hypersurface complexe. *Ann. Inst. Fourier, Grenoble*, 23, 4 (1973), 261-270.
- [Lo] LOOIJENGA, E.J.N. *Isolated Singular Points on Complete Intersections*. LMS Lecture Notes 77, Cambridge Univ. Press 1984.
- [Mi] MILNOR, J. Singular points of complex hypersurfaces. *Ann. of Math. Studies* 61, Princeton 1968.
- [Ne] NÉMETHI, A. The zeta-function of singularities. *J. of Algebraic Geometry* 2 (1993), 1-23.
- [Sch] SCHRAUWEN, R. *Series of Singularities and Their Topology*. PhD thesis, University of Utrecht 1991.
- [Si] SIERSMA, D. The monodromy of a series of hypersurface singularities. *Comment. Math. Helvetici* 65 (1990), 181-197.
- [Ti] TIBĂR, M. *The Lefschetz Number of a Monodromy Transformation*. Dissertation, University of Utrecht 1992.

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