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- (b) a set defined as in (4) or — if case — a similar one in a carrousel disc of order 1, or
- (c) a centre of a carrousel disc of order 1 inside  $A_i$ , for some  $i \in P^{(1)}$ , or
- (d) the centre  $(0, \eta)$  of the big carrousel disc.

2.6. *Definition.* Let  $\mathcal{I}^{(1)}$  be a maximal set of indices  $i \in P^{(1)}$  such that, if  $i_1, i_2 \in \mathcal{I}^{(1)}$ , then  $\hat{C}_{i_1}^{(1)} \neq \hat{C}_{i_2}^{(1)}$ .

For any  $i \in \mathcal{I}^{(1)}$ , denote by  $\delta(i)$  the carrousel disc of order 1 centred at the point  $c(i) := \hat{C}_i^{(1)} \cap (D_\alpha \times \{\eta\})$ . Let  $a(i)$  be an arbitrarily chosen point on the boundary  $\partial\bar{\delta}(i)$ ; it is, by definition, a regular value for  $l_\alpha$ .

*Definition.* Let  $a \in (D_\alpha \setminus 0) \times \{\eta\}$  and let  $F'_a$  be the fibre of  $l_\alpha$  over  $a$ . If  $a$  is fixed by the carrousel, then the monodromy  $h_f$  restricts to an action on  $H^\bullet(F'_a)$ , denoted by  $h'_a$ .

With these notations, we may formulate the following:

2.7. THEOREM. *If  $f \in \mathfrak{m}_{\mathbf{x},0}$  and  $l \in \Omega_f$ , then:*

$$\Lambda(f) = \Lambda(f|_{\{l=0\}}) + \sum_{i \in \mathcal{I}^{(1)}} [\Lambda(h'_{c(i)}) - \Lambda(h'_{a(i)})].$$

*Proof.* The Lefschetz number  $\Lambda(f)$  splits into a sum, following the decomposition of the set of fixed points into connected components, see 2.5. We use a suitable open covering of a set defined as in (4) and then apply the Mayer-Vietoris exact sequence. The reason of considering the set  $\mathcal{I}^{(1)}$  relies on the above discussion. By a straightforward computation, using also Lemma 2.1, we get our formula.  $\square$

Notice that carrousel discs of order  $\geq 2$  do not enter in the above formula. For the computation of  $\Lambda(h'_{c(i)})$ ,  $\Lambda(h'_{a(i)})$ , we refer to Remarks 3.6. There will be an example at the end.

### 3. ZETA-FUNCTION AND CARROUSEL MONODROMIES

3.1. Loosely speaking, each “important point” of the carrousel disc is fixed after a finite number of turns of the carrousel. We have seen that the set of fixed points after one turn determines the Lefschetz number  $\Lambda(h_f)$ . So the set of fixed points after  $k$  turns is the one responsible for the number  $\Lambda(h_f^k)$ . It may contain a finite number of circles consisting of regular values for the map  $l_\alpha$ . Actually, these circles do not count, as shown by Lemma 2.1 (where

$h_f$  has to be replaced by  $h_f^k$ ). By examining the proof of Theorem 2.1, we get a slightly more general result:

PROPOSITION. Let  $k \geq 1$ . If  $n_{i,1} \nmid k, \forall i \in \{1, \dots, r\}$ , then  $\Lambda(h_f^k) = \Lambda(h_{f|\{l=0\}}^k)$ .  $\square$

3.2. *Definition.* Let  $U \subset D_\alpha \times \{\eta\}$  and let  $k_U := \min\{k \mid U \text{ is globally fixed by the } k^{\text{th}} \text{ iteration of the carousel}\}$ . Then  $k_f^{k_U}$  restricts to an action on  $H^\bullet(l_\alpha^{-1}(U))$ , which we denote by  $h'_U$ . We call such actions *carousel monodromies*.

3.3. The zeta-function is determined by the set of Lefschetz numbers  $\Lambda(h_f^k), k \geq 1$ , as follows (see e.g. [Mi, p. 77], [A'C-2, p. 234]). If the integers  $s_1, s_2, \dots$  are inductively defined by  $\Lambda(h_f^k) = \sum_{i|k} s_i, k \geq 1$ , then the zeta-function of  $f$  is given by:

$$(5) \quad \xi_f(t) = \prod_{i \geq 1} (1 - t^i)^{-s_i/i}.$$

On the other hand, if  $\mathcal{B}^{(k)}$  denotes some small enough neighbourhood of the set of fixed points of the  $k^{\text{th}}$  power of the carousel, then  $h_f^k$  acts on the cohomology  $H^\bullet(l_\alpha^{-1}(\mathcal{B}^{(k)}))$  and, with the definition above, we get  $\Lambda(h_f^k) = \Lambda(h'_{\mathcal{B}^{(k)}})$ .

Let's consider the annulus  $A_i$ , as before, in the big carousel disc. Denote by  $h_{A_i}$  the restriction of  $h_f$  to  $H^\bullet(l_\alpha^{-1}(A_i))$ .

If  $x \in A_i$  is fixed by some power  $k$  of the carousel, then this power has to be a multiple of  $n_{i,1}$ . This remark and formula (5) yield the relation:

$$(6) \quad [\zeta_{h_{A_i}}(t)]^{n_{i,1}} = \zeta_{h_{A_i}^{n_{i,1}}}(t^{n_{i,1}}).$$

*Definition.* For any  $i \in \{1, \dots, r\}$ , denote by  $\delta(i)^{(1)}$  the carousel disc of order 1 centred at an arbitrarily chosen point of  $\hat{C}_i^{(1)} \cap (D_\alpha \times \{\eta\})$ , but fixed once and for all.

Let  $\mathcal{L}^{(1)} := \{\delta = \delta(i)^{(1)} \mid i \in \{1, \dots, r\}, \delta(i)^{(1)} \text{ is not contained in any other carousel disc of order 1}\}$ . For  $\delta \in \mathcal{L}^{(1)}$ , denote by  $a(\delta)$  an arbitrarily chosen point on the boundary  $\partial\bar{\delta}$ .

Then we have the next recursive formula:

$$3.4. \text{ THEOREM. } \zeta_f(t) = \zeta_{f|\{l=0\}}(t) \cdot \prod_{\delta \in \mathcal{L}^{(1)}} \zeta_{h'_\delta}(t^{n_{i,1}}) \cdot \zeta_{h_{a(\delta)}^{-1}}(t^{n_{i,1}}).$$

*Proof.* We apply Mayer-Vietoris exact sequences to the covering of the carousel disc described before. Since the fixed circles do not count for the

Lefschetz numbers, we get that the zeta-function is a product over all different annuli, each factor being of the form  $\zeta_{h_{A_i}}(t)$ .

We employ the notations in 2.5. Notice that the set  $\mathcal{K}_i^{(1)}$  is well defined for any  $i \in \{1, \dots, r\}$ . One can easily show that  $A_i$  retracts to the subset  $\mathcal{R}_i := S_i \cup \bigcup_{\delta \in \mathcal{K}_i^{(1)}} \delta$ , hence  $\zeta_{h_{A_i}^{n_{i,1}}}(t) = \zeta_{h'_{\mathcal{R}_i}}(t)$ .

If  $\delta \in \mathcal{K}_i^{(1)}$ , then notice that there are  $n_{i,1}$  carrousel discs in  $A_i$  of the same radius as  $\delta$ ; if  $\delta_1, \delta_2$  are any two of them, then  $\zeta_{h'_{\delta_1}}(t) = \zeta_{h'_{\delta_2}}(t)$ .

An open covering of  $\mathcal{R}_i$  and a Mayer-Vietoris argument lead to the conclusion:

$$\zeta_{h'_{\mathcal{R}_i}}(t) = \prod_{\delta \in \mathcal{L}^{(1)}} [\zeta_{h'_\delta}(t)]^{n_{i,1}} \cdot [\zeta_{h'_{a(\delta)}}(t)]^{n_{i,1}}.$$

Using (6), our formula is now proved. Notice that the factor  $\zeta_{f_{\{l=0\}}}(t)$  corresponds to the disc  $A_0$  defined in 1.8.  $\square$

It is not hard to figure out how the process started in the proof above may continue. We apply Theorem 3.4 with  $h_f$  replaced by  $h'_\delta$  and get a formula for the zeta-function  $\zeta_{h'_\delta}(t)$ , for any  $\delta \in \mathcal{L}^{(1)}$ . In a finite number of steps, going through the carrousel discs of order  $1, 2, \dots, m$ , where  $m := \max\{g_i \mid i \in \{1, \dots, r\}\}$ , we get a formula for  $\zeta_f(t)$ . To write it down, we need just the following notations.

*Definition.* Let  $\delta(i)^{(k)}$  denote the carrousel disc of order  $k$  centred at a fixed (arbitrarily chosen) point of the set  $\hat{C}_i^{(k)} \cap (D_\alpha \times \{\eta\})$ . (This later set contains exactly  $n_{i,1} \cdots n_{i,k}$  points). Denote  $\mathcal{C}(\Delta') := \{\delta(i)^{(k)} \mid i \in \{1, \dots, r\}, k \in \{1, \dots, m\}\}$ .

For any  $\delta \in \mathcal{C}(\Delta')$ , denote by  $c(\delta)$  its centre and by  $a(\delta)$  an arbitrarily chosen point on  $\partial\bar{\delta}$ .

Let  $\delta \in \mathcal{C}(\Delta')$ , where  $\delta = \delta(i)^{(k)}$ , for some indices  $i$  and  $k$  as above. Then define  $n(\delta) := n_{i,1} \cdots n_{i,k}$ .

Thus we get the following general zeta-function formula:

3.5. THEOREM.  $\zeta_f(t) = \zeta_{f_{\{l=0\}}}(t) \cdot \prod_{\delta \in \mathcal{C}(\Delta')} \zeta_{h'_{c(\delta)}}(t^{n(\delta)}) \cdot \zeta_{h'_{a(\delta)}}^{-1}(t^{n(\delta)}).$   $\square$

By using a decreasing induction,  $\zeta_f(t)$  will become finally a product of integer powers of cyclotomic polynomials.

3.6. *Remarks.* (a) The points  $a(\delta), \delta \in \mathcal{C}(\Delta')$  may also be defined as follows (the precise details are left to the reader):

Let  $\delta = \delta(i)^{(k)}$  and let  $\hat{C}_i^{(k)}$  be (formally) defined by the equation (see (3)):  $u_i = a_{k_i} \lambda^{m_{i,1}/n_{i,1}} + \dots + \sum_{l=1}^{l_k} b_{k,l} \lambda^{(m_k+l)/n_{i,1} \dots n_{i,k}}$ . Then define a curve  $G_{i,k}$ , by slightly perturbing in this equation just the last coefficient  $b_{k,l_k}$ , such that  $G_{i,k} \neq \hat{C}_j^{(k)}$ ,  $\forall j \in \{1, \dots, r\}$ . For  $k = g_i$ , we cut the Puiseux expansion at a sufficiently high power of  $\lambda$  and modify the last coefficient. It follows that  $a(\delta(i)^{(k)})$  may be identified to the point in  $G_{i,k} \cap (D_\alpha \times \{\eta\})$  which is in the closest neighbourhood of  $c(\delta(i)^{(k)})$ .

(b) Let  $\delta := \delta(i)^{(k)}$ . Then  $c(\delta)$  is a regular value for the map  $l_\alpha$  if and only if, for any  $j \in \{1, \dots, r\}$  such that  $\hat{C}_j^{(k)} = \hat{C}_i^{(k)}$ , we have  $g_j > k$ . It is possible that  $a(\delta)$  cannot be chosen arbitrarily close to  $c(\delta)$ , see also Remark 1.6.

(c) The carrousel monodromies  $h'_{c(\delta)}, h'_{a(\delta)}$  may be defined as monodromies of functions. This remark was used by Lê in his proof of the Monodromy Theorem [Lê-1], see also [Lo]. For instance, if  $\delta = \delta(i)^{(k)}$  and  $(u_i^{(k)}(t), \lambda(t))$  is the parametrization of  $\hat{C}_i^{(k)}$  defined in 1.5, then the pull-back diagram:

$$(7) \quad \begin{array}{ccc} (\mathbf{X}_i^{(k)}, 0) & \rightarrow & (\mathbf{X}, 0) \\ f_i^{(k)} \downarrow & & \downarrow \Phi \\ (\mathbf{C}, 0) & \xrightarrow{(u_i^{(k)}, \lambda)} & (\mathbf{C}^2, 0) \end{array}$$

defines a space  $(\mathbf{X}_i^{(k)}, 0)$  and a function  $f_i^{(k)}$  on it. Then  $h'_{c(\delta)}$  is the monodromy of  $f_i^{(k)}$ .

3.7. We illustrate the formula on the following particular case: *any component  $\Delta_i$  has just one Puiseux pair*, i.e.  $g_i = 1$ ,  $\forall i \in \{1, \dots, r\}$ . We assume, for the sake of simplicity, that the sets of components of  $\Gamma(l, f)$  and  $\Delta(l, f)$  are in one-to-one correspondence.

In this case, we have  $\hat{C}_i^{(1)} = \Delta_i$  and a carrousel disc  $\delta(i)^{(1)}$  is an arbitrarily small disc centred at  $c(\delta(i)^{(1)}) \in \Delta_i \cap (D_\alpha \times \{\eta\})$ , which is pointwise fixed by the  $n_{i,1}$ <sup>th</sup> iterate of the big carrousel. It follows that the point  $a(\delta(i)^{(1)})$  can be chosen arbitrarily close to  $c(\delta(i)^{(1)})$ . The centres  $c(\delta)$ ,  $\delta \in \mathcal{C}(\Delta')$  are critical values for the map  $l_\alpha$ . Let  $c(i)$  denote a fixed, arbitrarily chosen point of the set  $\Delta_i \cap (D_\alpha \times \{\eta\})$ . Then  $\mathcal{C}(\Delta')$  can be identified to the set  $\{c(i) \mid i \in \{1, \dots, r\}\}$ . With these notations, the zeta-function formula becomes

$$(8) \quad \zeta_f(t) = \zeta_{f|_{\{l=0\}}}(t) \cdot \prod_{i \in \{1, \dots, r\}} \zeta_{h'_{c(i)}}^{\text{rel}}(t^{n_{i,1}}),$$

where  $h_{c(i)}^{\text{rel}}: H^\bullet(l_\alpha^{-1}(c(\delta)), l_\alpha^{-1}(a(\delta))) \hookrightarrow$  is the relative monodromy and its zeta-function is  $\zeta_{h_{c(i)}^{\text{rel}}}(t) = \zeta_{h'_{c(\delta)}}(t) \zeta_{h'_{a(\delta)}}^{-1}(t)$ . One also gets  $\Lambda(f) = \Lambda(f|_{\{l=0\}}) + \sum_{i \in \{1, \dots, r\}, n_{i,1} = 1} \Lambda(h_{c(i)}^{\text{rel}})$ .

By standard arguments,  $H^\bullet(l_\alpha^{-1}(c(\delta)), l_\alpha^{-1}(a(\delta)))$  is isomorphic to the direct sum of reduced cohomologies  $\bigoplus_{v \in l_\alpha^{-1}(c(\delta)) \cap \Gamma} \tilde{H}^{\bullet-1}(F'_v)$ , where  $F'_v := B_{v,\varepsilon} \cap l_\alpha^{-1}(a(\delta))$  is the local Milnor fibre and  $B_{v,\varepsilon}$  is a Milnor ball of the isolated singularity at  $v$ . Let  $d_i := \# l_\alpha^{-1}(c(i)) \cap \Gamma$ .

A point  $v \in l_\alpha^{-1}(c(i)) \cap \Gamma$  goes, after  $n_{i,1}$  complete turns of the carrousel, to  $v' \in l_\alpha^{-1}(c(i)) \cap \Gamma$  and  $v' \neq v$  if  $n_{i,1} \geq 2$ . After exactly  $n_{i,1}d_i$  turns, the point  $v$  is fixed.

It becomes clear how the relative monodromy acts on the above direct sum; by similar arguments as those in [Si, p. 192], one shows that the matrix of  $h_{c(i)}^{\text{rel}}$  may be assumed to have the following block decomposition

$$\begin{bmatrix} 0 & 0 & \dots & \dots & 0 & \mathbf{V}_i \mathbf{T}_i^{n_{i,1}d_i} \\ \mathbf{I} & 0 & \dots & \dots & 0 & 0 \\ 0 & \mathbf{I} & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & 0 & \dots \\ 0 & \dots & \dots & \dots & 0 & \mathbf{I} & 0 \end{bmatrix},$$

where, at some fixed  $v(i) \in l_\alpha^{-1}(c(i))$ ,  $\mathbf{I}$  is the identity matrix on  $\tilde{H}^\bullet(F'_{v(i)})$ ,  $\mathbf{T}_i$  is the *horizontal monodromy* of the transversal singularity and  $\mathbf{V}_i$  is the *vertical monodromy* of the local system on  $\Gamma_i \setminus \{0\}$ , with fibre  $\tilde{H}^\bullet(F'_{v(i)})$ . Then  $\zeta_{h_{c(i)}^{\text{rel}}}(t) = \prod_{j \geq 0} \det[\mathbf{I} - t^{d_i} \mathbf{V}_i \mathbf{T}_i^{n_{i,1}d_i}; \tilde{H}^j(F'_{v(i)})]^{(-1)^j}$ . Finally, our formula looks as follows:

$$(9) \quad \zeta_f(t) = \zeta_{f|_{\{l=0\}}}(t) \cdot \prod_{i \in \{1, \dots, r\}} \prod_{j \geq 0} \det[\mathbf{I} - t^{n_{i,1}d_i} \mathbf{V}_i \mathbf{T}_i^{n_{i,1}d_i}; \tilde{H}^j(F'_{v(i)})]^{(-1)^j}.$$

3.8. This latter one may be easily specialized to the Siersma's formula [loc. cit.]. Let  $A_m$  be the most exterior annulus and assume that the components of  $\Delta$  which cut  $A_m$  are  $\Delta_1, \dots, \Delta_s$  and they have just one Puiseux pair. Denote  $D_{m-1} := D_\alpha \times \{\eta\} \setminus A_m$ . By our approach we get  $\zeta_f(t) = \zeta_{h_{D_{m-1}}}(t) \cdot \prod_{i \in \{1, \dots, s\}} \zeta_{h_{c(i)}^{\text{rel}}}(t^{n_{i,1}})$ .

Let then  $g$  be a function with 1-dimensional singular locus  $\Sigma = \cup_{i \in \{1, \dots, s\}} \Sigma_i$  and let  $f := g + l^K$ , for some  $l \in \Omega_g$ , with  $K \in \mathbf{N}$  large enough. Then  $f$  is an isolated singularity and, as shown in [Si], one may identify the monodromy of the Milnor fibre  $F_g$  to  $h_{D_{m-1}}$ . The degree of the covering  $\Sigma_i \setminus \{0\} \rightarrow \Delta_i \setminus \{0\}$  is  $d_i$ . Then one gets [Si, p. 183]:

$$(10) \quad \zeta_f(t) = \zeta_g(t) \cdot \prod_{i \in \{1, \dots, s\}} \det[\mathbf{I} - t^{Kd_i} V_i \cdot T_i^{Kd_i}]^{(-1)^{\dim(\mathbf{X}, 0)}}.$$

3.9. *Example.* Let  $\mathbf{X} := \{x^3 + y^4 + z^3 = 0\} \subset \mathbf{C}^3$  and let  $f \in \mathfrak{m}_{\mathbf{X},0}$  be the function induced by  $\tilde{f} \in \mathfrak{m}_{\mathbf{C}^3,0}$ ,  $\tilde{f} = x$ . Consider the linear function  $l$  induced by  $\tilde{l} = y$ . Then  $l \in \Omega_f$ . We get that  $\Delta(l, f)$  is irreducible and has the Puiseux parametrization:  $l = \alpha v^3$ ,  $\lambda = v^4$ , where  $\alpha$  is a nonzero constant, easy to determine.

Let  $c \in \Delta(l, f) \cap (D_\alpha \times \{\eta\})$  and let  $a \notin \Delta(l, f) \cap (D_\alpha \times \{\eta\})$  be a neighbour point of  $c$ .

The monodromy  $h'_a$  can be identified to the monodromy of the function  $f_a: (\mathbf{X}_a, 0) \rightarrow (\mathbf{C}, 0)$  induced by  $\tilde{f}_a = v$ , where  $\mathbf{X}_a := \{x = v^4, y = v^3, z = \sqrt[3]{2\gamma v^4}\}$  and  $\gamma$  is a 3-root of  $-1$ . We get  $\zeta_{h'_a}(t) = (1-t)^{-3}$ , hence  $\zeta_{h_c^{\text{rel}}} = (1-t)^2$ .

By using (8), the final result is  $\zeta_f(t) = (1-t)^{-3}(1-t^4)^2$ . We also get  $\Lambda(f) = 3$ .

Notice that there is another way of computing the zeta function in this example, by using the usual  $\mathbf{C}^*$ -action on  $\mathbf{X}$ , which fixes the zero set  $\{\tilde{f} = 0\}$ . It follows that the monodromy  $h_f$  of  $f$  is equal to the 3<sup>rd</sup> power of the monodromy  $h_g$  of the function  $g: (\mathbf{C}^2, 0) \rightarrow (\mathbf{C}, 0)$ ,  $g = y^4 + z^3$  and  $\zeta_{h_g^3}(t)$  can be computed from the eigenvalues of  $h_g$ . If we change the above function  $\tilde{f}$  into  $\tilde{f}_1 := x + y$ , then the set  $\{\tilde{f}_1 = 0\}$  is no more invariant under the above-mentioned  $\mathbf{C}^*$ -action. The computations for the zeta-function of  $h_{f_1}$  are slightly more complicated, since we get two Puiseux pairs, with  $n_{1,1} = 1$ ,  $n_{1,2} = 3$ . This time, the result is  $\zeta_{f_1}(t) = (1-t)^{-1}(1-t^3)^{-1}(1-t^9)$ .

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