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Fix $p \geqslant 0$ and consider the Serre spectral sequence for the fibration $S^k \to E \to B_0$ with coefficients in $\mathbf{F}_p$. If this fails to collapse then $H^k(\pi_0): H^k(B_0; \mathbf{F}_p) \to H^k(E; \mathbf{F}_p)$ is surjective. Since $l > k$ it is always true that $H^k(\pi_1)$ is surjective. Choose classes $\alpha \in H^k(B_0; \mathbf{F}_p)$, $\beta \in H^k(B_1; \mathbf{F}_p)$ mapping to the same non-zero class in $H^k(E; \mathbf{F}_p)$. The Mayer-Vietoris sequence for the decomposition $S^{n+1} = D_0 \underset{E}{\cup} D_1$ then gives a class $\gamma \in H^k(S^{n+1}; \mathbf{F}_p)$ restricting to α and β, which is absurd.

Thus the spectral sequence for $S^k \to E \to B_0$ collapses and so $H_*(B_0; \mathbf{F}_p) \cong H_*(S^l \times S^{l+k}; \mathbf{F}_p)$. Using Poincaré duality for B_0 we see that $H^*(B_0; \mathbf{F}_p)$ and $H^*(S^l \times S^{l+k}; \mathbf{F}_p)$ are isomorphic as graded algebras. Thus B_0 is elliptic by 3.4 and E is elliptic by 3.3.

3.6. Simply connected closed manifolds M with a smooth action by a compact Lie group G, having a simply connected codimension one orbit.

Here we may assume G is connected. Let the orbit be G/K, and convert the inclusion of G/K into a fibration $F \to G/K \to M$. From [9; Table 1.5] we see that for any p, $\dim H_i(F; \mathbf{F}_p) \leqslant 2$, all i. Thus applying the Serre spectral sequence to the fibration $\Omega(G/K) \to \Omega M \to F$ and using 3.1 for G/K we see that $H_*(\Omega M; \mathbf{F}_p)$ grows polynomially.

3.7. Simply connected manifolds $M \# N$ with each of the rings $H^*(M; \mathbf{Z})$, $H^*(N; \mathbf{Z})$ generated by a single class.

By Van Kampen's theorem both M and N are simply connected, and so their fundamental cohomology classes are not torsion. Since each ring is monogenic, $H^*(M; \mathbf{Z})$ and $H^*(N; \mathbf{Z})$ are torsion free. Thus $H^*(M; \mathbf{F}_p)$ and $H^*(N; \mathbf{F}_p)$ are also monogenic, and so $H^*(M \# N; \mathbf{F}_p)$ is doubly generated. Now apply 3.4.

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