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Fix  $p \geq 0$  and consider the Serre spectral sequence for the fibration  $S^k \rightarrow E \rightarrow B_0$  with coefficients in  $\mathbf{F}_p$ . If this fails to collapse then  $H^k(\pi_0): H^k(B_0; \mathbf{F}_p) \rightarrow H^k(E; \mathbf{F}_p)$  is surjective. Since  $l > k$  it is always true that  $H^k(\pi_1)$  is surjective. Choose classes  $\alpha \in H^k(B_0; \mathbf{F}_p)$ ,  $\beta \in H^k(B_1; \mathbf{F}_p)$  mapping to the same non-zero class in  $H^k(E; \mathbf{F}_p)$ . The Mayer-Vietoris sequence for the decomposition  $S^{n+1} = D_0 \cup_E D_1$  then gives a class  $\gamma \in H^k(S^{n+1}; \mathbf{F}_p)$  restricting to  $\alpha$  and  $\beta$ , which is absurd.

Thus the spectral sequence for  $S^k \rightarrow E \rightarrow B_0$  collapses and so  $H_*(B_0; \mathbf{F}_p) \cong H_*(S^l \times S^{l+k}; \mathbf{F}_p)$ . Using Poincaré duality for  $B_0$  we see that  $H^*(B_0; \mathbf{F}_p)$  and  $H^*(S^l \times S^{l+k}; \mathbf{F}_p)$  are isomorphic as graded algebras. Thus  $B_0$  is elliptic by 3.4 and  $E$  is elliptic by 3.3.

3.6. *Simply connected closed manifolds  $M$  with a smooth action by a compact Lie group  $G$ , having a simply connected codimension one orbit.*

Here we may assume  $G$  is connected. Let the orbit be  $G/K$ , and convert the inclusion of  $G/K$  into a fibration  $F \rightarrow G/K \rightarrow M$ . From [9; Table 1.5] we see that for any  $p$ ,  $\dim H_i(F; \mathbf{F}_p) \leq 2$ , all  $i$ . Thus applying the Serre spectral sequence to the fibration  $\Omega(G/K) \rightarrow \Omega M \rightarrow F$  and using 3.1 for  $G/K$  we see that  $H_*(\Omega M; \mathbf{F}_p)$  grows polynomially.

3.7. *Simply connected manifolds  $M \# N$  with each of the rings  $H^*(M; \mathbf{Z})$ ,  $H^*(N; \mathbf{Z})$  generated by a single class.*

By Van Kampen's theorem both  $M$  and  $N$  are simply connected, and so their fundamental cohomology classes are not torsion. Since each ring is monogenic,  $H^*(M; \mathbf{Z})$  and  $H^*(N; \mathbf{Z})$  are torsion free. Thus  $H^*(M; \mathbf{F}_p)$  and  $H^*(N; \mathbf{F}_p)$  are also monogenic, and so  $H^*(M \# N; \mathbf{F}_p)$  is doubly generated. Now apply 3.4.

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