

**Zeitschrift:** L'Enseignement Mathématique  
**Herausgeber:** Commission Internationale de l'Enseignement Mathématique  
**Band:** 39 (1993)  
**Heft:** 3-4: L'ENSEIGNEMENT MATHÉMATIQUE

**Artikel:** CARROUSEL MONODROMY AND LEFSCHETZ NUMBER OF SINGULARITIES  
**Autor:** Tibr, Mihai  
**Kapitel:** Introduction  
**DOI:** <https://doi.org/10.5169/seals-60425>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 15.03.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

## CARROUSEL MONODROMY AND LEFSCHETZ NUMBER OF SINGULARITIES

by Mihai TIBĂR

### INTRODUCTION

Let  $f: (\mathbf{X}, x) \rightarrow (\mathbf{C}, 0)$  be a holomorphic function on an analytic germ  $(\mathbf{X}, x)$ . Let  $h_f$  denote the monodromy of the germ  $\Psi_f^\bullet(\mathbf{C}_\mathbf{X}^\bullet)_x$  of neighbouring cycles. One defines its *Lefschetz number*

$$\Lambda(h_f) := \sum_{i \geq 0} (-1)^i \text{trace} [h_f; \Psi_f^i(\mathbf{C}_\mathbf{X}^\bullet)_x],$$

and its *zeta-function*

$$\zeta_{h_f}(t) := \prod_{i \geq 0} \det [I - t \cdot h_f; \Psi_f^i(\mathbf{C}_\mathbf{X}^\bullet)_x]^{(-1)^{i+1}}.$$

We alternatively denote them by  $\Lambda(f)$ , respectively  $\zeta_f(t)$ .

A theorem of Eisenbud and Neumann [EN, Theorem 4.3] asserts that the zeta-function of a *fibred multilink*  $L$  is the product of the zeta-functions over all *splice components* of  $L$ . If the multilink is defined by some Cerf diagram  $\Delta(l, f)$ , then  $\zeta_f(t)$  becomes the zeta-function of the multilink  $L$ , this time with coefficients in a local system. This observation of Némethi [Ne] enables him to prove an inductive formula for  $\zeta_f(t)$ , in terms of invariants of the so called EN-diagram (splice diagram); compare to the one of Eisenbud and Neumann [EN, p. 96]. Some quite strong results in the 3-dimensional link theory are involved in the proofs.

Our approach is based on Lê's carousel construction and is therefore more geometric and selfcontained. It yields inductive formulae for  $\Lambda(f)$  and  $\zeta_f(t)$  directly from the Puiseux parametrization of  $\Delta(l, f)$ . Moreover, it clarifies the contribution, however essential in general, of the "nonessential" terms in this parametrization — which may be not clear from the definition of the splice diagram of an algebraic link given in [EN, p. 53], simply because such terms are completely omitted. One can therefore compare to our definitions 1.5 ÷ 7.

The formula for  $\zeta_f(t)$  will be not the same, but quite similar to the ones before. The ingredients are zeta-functions of fibres over certain periodic points in the carousel disc. We show in Sections 2 and 3 how to define these points from the Puiseux expansion of  $\Delta(l, f)$ . We end by some applications.

*Acknowledgement.* This work is based on a piece of the author's dissertation [Ti]. He much benefited from talks with Dirk Siersma, whose paper [Si] incited him to do this research (see 3.8).

## 1. THE CARROUSEL REVISITED

1.1. We first briefly recall the carousel construction, following closely [Lê-1] and [Lê-3], then give the necessary definitions for our study. One regards  $(\mathbf{X}, x)$  as being embedded in  $(\mathbf{C}^N, 0)$ , for some sufficiently large  $N \in \mathbf{N}$ . We assume that, unless otherwise stated, all the irreducible components of  $(\mathbf{X}, 0)$  have dimensions greater than 1.

Let  $\mathcal{L}$  be a small enough representative of  $(\mathbf{X}, 0)$ . Let  $\Gamma(l, f)$  be the *polar curve* of  $f$  with respect to a linear function  $l: (\mathbf{X}, 0) \rightarrow (\mathbf{C}, 0)$ , relatively to a fixed *Whitney stratification*  $\mathcal{S}$  on  $\mathcal{L}$  which satisfies *Thom condition*  $(a_f)$ .

The polar curve  $\Gamma(l, f)$  exists for a Zariski open subset  $\hat{\Omega}_f$  in the space of linear germs  $l: (\mathbf{C}^N, 0) \rightarrow (\mathbf{C}, 0)$ . If one does not impose  $\Gamma(l, f)$  to be reduced then one gets a larger set  $\Omega_f \supset \hat{\Omega}_f$  which is sometimes useful to deal with (see e.g. Example 2.2). (We only mention that one can enlarge even  $\Omega_f$ : modify its definition by allowing also nonlinear functions.)

1.2. Let  $l \in \Omega_f$  and let  $\Phi := (l, f): (\mathbf{X}, 0) \rightarrow (\mathbf{C}^2, 0)$ . We denote by  $(u, \lambda)$  the pair of coordinates on  $\mathbf{C}^2$ .

The curve germ (with reduced structure)  $\Delta(l, f) := \Phi(\Gamma(l, f))$  is called the *Cerf diagram* (of  $f$  with respect to  $l$ , relative to  $\mathcal{S}$ ). We shall use the same notation  $\Gamma(l, f)$ , respectively  $\Delta(l, f)$  for suitable representatives of these germs.

There is a fundamental system of “privileged” open polydiscs in  $\mathbf{C}^N$ , centred at 0, of the form  $(D_\alpha \times P_\alpha)_{\alpha \in A}$  and a corresponding fundamental system  $(D_\alpha \times D'_\alpha)_{\alpha \in A}$  of 2-discs at 0 in  $\mathbf{C}^2$ , such that  $\Phi$  induces, for any  $\alpha \in A$ , a topological fibration

$$\begin{aligned} \Phi_\alpha: \mathcal{L} \cap (D_\alpha \times P_\alpha) \cap \Phi^{-1}(D_\alpha \times D'_\alpha \setminus (\Delta(l, f) \cup \{\lambda = 0\})) \\ \rightarrow D_\alpha \times D'_\alpha \setminus (\Delta(l, f) \cup \{\lambda = 0\}). \end{aligned}$$