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| <b>Autor:</b>       | Tibr, Mihai   |
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## CARROUSEL MONODROMY AND LEFSCHETZ NUMBER OF SINGULARITIES

by Mihai TIBĂR

### INTRODUCTION

Let  $f: (\mathbf{X}, x) \rightarrow (\mathbf{C}, 0)$  be a holomorphic function on an analytic germ  $(\mathbf{X}, x)$ . Let  $h_f$  denote the monodromy of the germ  $\Psi_f^\bullet(\mathbf{C}_x^\bullet)_x$  of neighbouring cycles. One defines its *Lefschetz number*

$$\Lambda(h_f) := \sum_{i \geq 0} (-1)^i \operatorname{trace} [h_f; \Psi_f^i(\mathbf{C}_x^\bullet)_x],$$

and its *zeta-function*

$$\zeta_{h_f}(t) := \prod_{i \geq 0} \det[I - t \cdot h_f; \Psi_f^i(\mathbf{C}_x^\bullet)_x]^{(-1)^{i+1}}.$$

We alternatively denote them by  $\Lambda(f)$ , respectively  $\zeta_f(t)$ .

A theorem of Eisenbud and Neumann [EN, Theorem 4.3] asserts that the zeta-function of a *fibred multilink*  $L$  is the product of the zeta-functions over all *splice components* of  $L$ . If the multilink is defined by some Cerf diagram  $\Delta(l, f)$ , then  $\zeta_f(t)$  becomes the zeta-function of the multilink  $L$ , this time with coefficients in a local system. This observation of Némethi [Ne] enables him to prove an inductive formula for  $\zeta_f(t)$ , in terms of invariants of the so called EN-diagram (splice diagram); compare to the one of Eisenbud and Neumann [EN, p. 96]. Some quite strong results in the 3-dimensional link theory are involved in the proofs.

Our approach is based on Lê's carrousel construction and is therefore more geometric and selfcontained. It yields inductive formulae for  $\Lambda(f)$  and  $\zeta_f(t)$  directly from the Puiseux parametrization of  $\Delta(l, f)$ . Moreover, it clarifies the contribution, however essential in general, of the “nonessential” terms in this parametrization — which may be not clear from the definition of the splice diagram of an algebraic link given in [EN, p. 53], simply because such terms are completely omitted. One can therefore compare to our definitions 1.5 ÷ 7.

The formula for  $\zeta_f(t)$  will be not the same, but quite similar to the ones before. The ingredients are zeta-functions of fibres over certain periodic points in the carrousel disc. We show in Sections 2 and 3 how to define these points from the Puiseux expansion of  $\Delta(l, f)$ . We end by some applications.

*Acknowledgement.* This work is based on a piece of the author's dissertation [Ti]. He much benefited from talks with Dirk Siersma, whose paper [Si] incited him to do this research (see 3.8).

## 1. THE CARROUSEL REVISITED

1.1. We first briefly recall the carrousel construction, following closely [Lê-1] and [Lê-3], then give the necessary definitions for our study. One regards  $(\mathbf{X}, x)$  as being embedded in  $(\mathbf{C}^N, 0)$ , for some sufficiently large  $N \in \mathbf{N}$ . We assume that, unless otherwise stated, all the irreducible components of  $(\mathbf{X}, 0)$  have dimensions greater than 1.

Let  $\mathcal{H}$  be a small enough representative of  $(\mathbf{X}, 0)$ . Let  $\Gamma(l, f)$  be the *polar curve* of  $f$  with respect to a linear function  $l: (\mathbf{X}, 0) \rightarrow (\mathbf{C}, 0)$ , relatively to a fixed *Whitney stratification*  $\mathcal{S}$  on  $\mathcal{H}$  which satisfies *Thom condition* ( $a_f$ ).

The polar curve  $\Gamma(l, f)$  exists for a Zariski open subset  $\hat{\Omega}_f$  in the space of linear germs  $l: (\mathbf{C}^N, 0) \rightarrow (\mathbf{C}, 0)$ . If one does not impose  $\Gamma(l, f)$  to be reduced then one gets a larger set  $\Omega_f \supset \hat{\Omega}_f$  which is sometimes useful to deal with (see e.g. Example 2.2). (We only mention that one can enlarge even  $\Omega_f$ : modify its definition by allowing also nonlinear functions.)

1.2. Let  $l \in \Omega_f$  and let  $\Phi := (l, f): (\mathbf{X}, 0) \rightarrow (\mathbf{C}^2, 0)$ . We denote by  $(u, \lambda)$  the pair of coordinates on  $\mathbf{C}^2$ .

The curve germ (with reduced structure)  $\Delta(l, f) := \Phi(\Gamma(l, f))$  is called the *Cerf diagram* (of  $f$  with respect to  $l$ , relative to  $\mathcal{S}$ ). We shall use the same notation  $\Gamma(l, f)$ , respectively  $\Delta(l, f)$  for suitable representatives of these germs.

There is a fundamental system of “privileged” open polydiscs in  $\mathbf{C}^N$ , centred at 0, of the form  $(D_\alpha \times P_\alpha)_{\alpha \in A}$  and a corresponding fundamental system  $(D_\alpha \times D'_\alpha)_{\alpha \in A}$  of 2-discs at 0 in  $\mathbf{C}^2$ , such that  $\Phi$  induces, for any  $\alpha \in A$ , a topological fibration

$$\begin{aligned}\Phi_\alpha: \mathcal{H} \cap (D_\alpha \times P_\alpha) \cap \Phi^{-1}(D_\alpha \times D'_\alpha \setminus (\Delta(l, f) \cup \{\lambda = 0\})) \\ \rightarrow D_\alpha \times D'_\alpha \setminus (\Delta(l, f) \cup \{\lambda = 0\}).\end{aligned}$$