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SMOOTHLY EMBEDDED 2-SPHERES AND EXOTIC 4-MANIFOLDS

by Paolo LISCA

1. INTRODUCTION

We prove two independent results concerning multiple smooth structures on certain topological 4-manifolds with boundary (theorems 1.1 and 1.2). The second of our results has been already proven by Akbulut [A]; we give here a different proof. The common underlying theme consists of obstructions to represent 2-homology classes of 4-manifolds by smoothly embedded spheres. Indeed, we use the fact that certain classes of self-intersection -1 or -2 cannot be represented by smoothly embedded spheres inside certain 4-manifolds, while they can inside others.

With our first result we give a partial answer to the following question, raised in [G]: given the 4-manifold with boundary $M(k, l, m)$, obtained by attaching 2-handles to B^4 according to the framed link of figure 1, how many diffeomorphism types are realized by permuting the integers k, l , and m ?

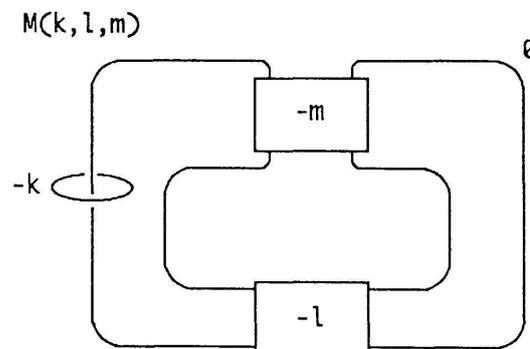


FIGURE 1

(In such pictures, a box with $-m$ inside means $-m$ full twists. A $-k$ next to a loop means that the framing has linking number $-k$ with the loop.)

Notice the equality $M(k, l, m) = M(k, m, l)$, obtained by an obvious isotopy of the link. In section 2 we prove the following:

THEOREM 1.1. *Let $k, l, m, k \neq l$, be positive integers with either $k \leq 2$ or $l \leq 2$, and s a non-negative integer. If either (i) $s \neq 0$ or (ii) $s = 0$ and $k \equiv l \pmod{2}$, then the manifolds $M(k, l, m) \#^s \overline{\mathbf{C}\mathbf{P}}^2$ and $M(l, k, m) \#^s \overline{\mathbf{C}\mathbf{P}}^2$ are homeomorphic but their interiors are not diffeomorphic.*

We have as a corollary that for n odd $M(1, n, 1)$ and $M(n, 1, 1)$ are homeomorphic but not diffeomorphic, a result already obtained in [G] by a different method.

As for the second result, let $W_i, i = 1, 2$ be the 4-manifolds with boundary obtained via the framed knots $K_i, i = 1, 2$ of figure 2.

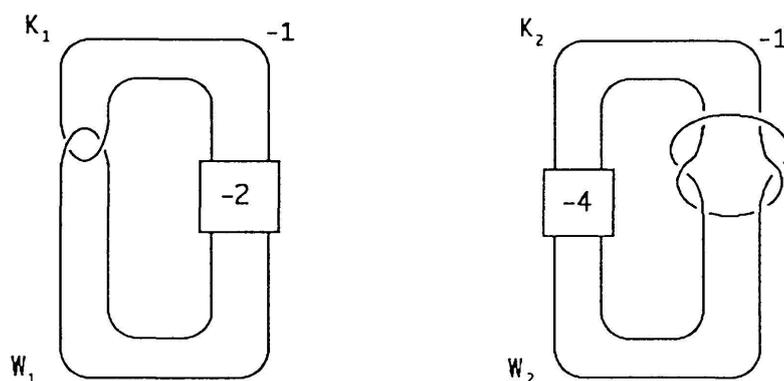


FIGURE 2

We give a new proof of the following theorem, due to Akbulut [A]:

THEOREM 1.2. *W_1 and W_2 are homeomorphic but their interiors are not diffeomorphic.*

Remark. In Akbulut's notation, the manifolds W_1 and W_2 are respectively $-Q_1$ and $-Q_2$.

To prove theorems 1.1 and 1.2 we use gauge-theoretical results about the smooth structure of blown-up elliptic surfaces [FM2]. We remark that also the analogous results from [G] and [A] depend on gauge theory.

Acknowledgments. We thank John Morgan for numerous discussions about embedded spheres in 4-manifolds, and for making available an early copy of his book with Bob Friedman [FM2]. We are also grateful to Riccardo Benedetti for helpful conversations.