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PROPOSITION 5.  $T'T'' = T''T'$  (see figure 9).

*Proof.* For any point of the annulus between  $\Gamma'$  and  $\Gamma$  there is an ellipse from the pencil through it. Both maps preserve this ellipse and both are shifts in the affine parameter on it. Since shifts commute, the proposition follows.

We refer to [T3] for a partial converse statement to the proposition.

One can also slightly generalize Poncelet's theorem (this generalization was known to Poncelet too). Consider a number of ellipses from a pencil:  $\Gamma, \Gamma', \Gamma''$  etc, and let  $\Gamma$  contain all the others. Choose a point  $x \in \Gamma$ , draw a tangent line to  $\Gamma'$ , find its intersection with  $\Gamma$ , draw a tangent line to  $\Gamma''$  etc. Then, if  $x$  returns back after a number of iterations, any initial point on  $\Gamma$  does. It follows again from the fact that  $T', T''$  etc. are shifts in the affine parameter on  $\Gamma$ .

We conclude with a conjecture. Let a smooth strictly convex curve  $\Gamma_0$  be given in the plane. Assume that its outer neighbourhood is foliated by convex curves  $\Gamma_\lambda, \lambda \in [0, \varepsilon[$ . Assume also that, for any line tangent to  $\Gamma_0$ , the (local) involution, defined on it by its intersections with the curves  $\Gamma_\lambda$ , is a projective transformation. Then the curves  $\Gamma_\lambda$  belong to a pencil of ellipses.

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