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PROPOSITION 5. $T'T'' = T''T'$ (see figure 9).

Proof. For any point of the annulus between Γ' and Γ there is an ellipse from the pencil through it. Both maps preserve this ellipse and both are shifts in the affine parameter on it. Since shifts commute, the proposition follows.

We refer to [T 3] for a partial converse statement to the proposition.

One can also slightly generalize Poncelet's theorem (this generalization was known to Poncelet too). Consider a number of ellipses from a pencil: Γ, Γ', Γ'' etc, and let Γ contain all the others. Choose a point $x \in \Gamma$, draw a tangent line to Γ', find its intersection with Γ, draw a tangent line to Γ'' etc. Then, if x returns back after a number of iterations, any initial point on Γ does. It follows again from the fact that T', T'' etc. are shifts in the affine parameter on Γ.

We conclude with a conjecture. Let a smooth strictly convex curve Γ_0 be given in the plane. Assume that its outer neighbourhood is foliated by convex curves Γ_λ, $\lambda \in [0, \varepsilon[$. Assume also that, for any line tangent to Γ_0, the (local) involution, defined on it by its intersections with the curves Γ_λ, is a projective transformation. Then the curves Γ_λ belong to a pencil of ellipses.

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REFERENCES

[A.-G.] ARNOLD, V. and A. GIVENTAL. *Symplectic Geometry*. Dynamical Syst. v.4. Springer (1990).

[B] BERGER, M. *Geometry*. Springer (1987).

[B.-K.-O.-R.] BOS, H., C. KERS, F. OORT and D. RAVEN. Poncelet's Closure Theorem. *Expos. Math. 5* (1987), 289-364.

[F.-T.] FUCHS, D. and S. TABACHNIKOV. Segments of Equal Areas. *Quantum, N2* (1992), translated from Kvant (in Russian).

[G.-H. 1] GRIFFITH, P. and J. HARRIS. A Poncelet Theorem in Space. *Comm. Math. Helv. 52* (1977), 145-160.

[G.-H. 2] GRIFFITH, P. and J. HARRIS. On Cayley's Explicit Solution of Poncelet's Porism. *L'Enseign. Math. 24* (1978), 31-40.

[M] MOSER, J. Is the Solar System Stable? *Math. Intell. 1, N2* (1978).

[T 1] TABACHNIKOV, S. On the Dual Billiard Problem. *Adv. Math.*, to appear.

[T 2] —— Dual Billiards. IHES preprint (1992).

[T 3] —— Commuting Dual Billiard Maps. ETH preprint (1993).

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