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Artikel: THE SUM OF THE CANTOR SET WITH ITSELF

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First consider the case where h has a unique ternary expansion, which is necessarily infinite. Then $S(k) = 2^{c(k)}$. To see, this, set $h = \sum a_n/3^n$ where $a_n = 0, 1,$ or 2 ($n = 1, 2, 3, \ldots$), $a_n \neq 0$ for infinitely many n and $a_n \neq 2$ for infinitely many n. We wish to count the number of representations $h = x + y$ where $x, y \in \frac{1}{2} C$; i.e., $x = \sum \varepsilon_n/3^n$, $y = \sum \varepsilon_n'/3^n$ and $\varepsilon_n, \varepsilon_n' = 0$ or 1. Now if $a_n = 0$, clearly $\varepsilon_n = \varepsilon_n' = 0$. Also if $a_n = 2$, $\varepsilon_n = \varepsilon_n' = 1$. However if $a_n = 1$, we can have $\varepsilon_n = 1$ and $\varepsilon_n' = 0$ or we can have $\varepsilon_n = 0$ and $\varepsilon_n' = 1$. Hence there are $2^{c(k)}$ choices for (x, y) (uncountable if $c(k)$ is infinite).

Next consider the case where h has two ternary expansions. Then they are necessarily of the form

$$h = .\, a_1 a_2 \ldots a_r 22 \ldots = .\, a_1 a_2 \ldots a_{r-1} b_r$$

where $a_1, a_2, \cdots, a_{r-1} = 0, 1, 2$, $a_r = 0$ or 1 and $b_r = a_r + 1$. Then using the ideas in the last paragraph and keeping in mind there are two counts (one for each representation of h), we have:

$$S(k) = \begin{cases} 3(2^{c(k)}) & \text{if} \quad a_r = 0\,. \\ 3(2^{c(k)-1}) & \text{if} \quad a_r = 1\,. \end{cases}$$

REFERENCES

[1] KOLMOGOROV, A.N. and S.V. FOMIN. *Introductory Real Analysis,* 1970, Prentice-Hall Publishers, p. 54.

[2] PAVONE, M. The Cantor set and a geometric construction. *L'Enseignement Mathématique,* 35 (1989), 41-49.

[3] SHALLIT, Jeffrey. Q785. *Mathematics Magazine,* 64 No. 5 (1991), 351 and 357.

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