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Autor: Felix, Yves / Halperin, Stephen / Thomas, Jean-Claude
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ELLIPTIC SPACES II

by Yves FELIX, Stephen HALPERIN [1]) and Jean-Claude THOMAS [2])

ABSTRACT. A simply connected finite CW complex X is *elliptic* if the homology of its loop space (coefficients in any field) grows at most polynomially. We show that in all other cases the loop space homology grows at least semi-exponentially, and we exhibit a number of geometrically interesting classes of spaces as elliptic, including: H spaces, homogeneous spaces, Poincaré duality complexes whose mod p cohomology is doubly generated (any p) and Dupin hypersurfaces in S^{n+1}.

1. INTRODUCTION

Let X be a simply connected finite CW complex, with loop space ΩX, and denote by $\mathbf{F}_p$, the prime field of characteristic p, p prime or zero. Our first main result asserts a dichotomy for the size of the loop space homology $H_*(\Omega X; \mathbf{F}_p)$:

THEOREM A. *Let X be a simply connected finite CW complex. For each p (prime or zero) there are exactly two possibilities: either*

(i) *There are constants $C > 0$ and $r \in \mathbf{N}$ such that*

$$\sum_{i=0}^{n} \dim H_i(\Omega X; \mathbf{F}_p) \leqslant Cn^r, \qquad n \geqslant 1 ,$$

Key words: loop space homology, depth, polynomial growth, Poincaré complex, elliptic, Dupin hypersurface.

AMS Mathematical subject classification: 55P35, 57P10, 57T25, 57S25, 53C25.

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