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COROLLARY 2 (C. Jordan [3]). *A primitive subgroup of $\text{Sym}(n)$ containing a transposition is all of $\text{Sym}(n)$.*

Proof. Let \mathcal{H} be a primitive subgroup of $\text{Sym}(n)$ and τ a transposition in \mathcal{H} . Then \mathcal{H} permutes the components Γ_i of $\Gamma(\mathcal{H}, \tau)$ and so the vertex sets V_i of the Γ_i are permuted by \mathcal{H} . The primitivity of \mathcal{H} implies that the set $\{1, 2, \dots, n\}$ can be partitioned into disjoint subsets permuted by \mathcal{H} only if each subset has order one or there is just one subset of order n . Since the vertex set of Γ_i has more than one element, there is only one component and $\mathcal{H} = \text{Sym}(n)$ by Corollary 1.

2. AN APPLICATION TO GALOIS THEORY

We extend the theorem mentioned in the introduction replacing the condition that the degree of the polynomial be a prime greater than 3 by the condition that the degree of the polynomial be divisible only by primes greater than 3.

THEOREM 2. *Let $f(x)$ be a polynomial of degree n with rational coefficients and irreducible over the rational field. Assume that $f(x)$ has exactly $n - 2$ real roots. If n is divisible only by primes greater than 3 then the Galois group of the splitting field of $f(x)$ is not solvable and $f(x)$ is not solvable by radicals.*

Proof. Let \mathcal{H} be the Galois group of $f(x)$ over the rational field. We view \mathcal{H} as a permutation group on the n roots of f . Then complex conjugation, τ , is a transposition in \mathcal{H} of the two nonreal roots. Since $f(x)$ is irreducible, \mathcal{H} is transitive on the set of n roots. By theorem 1, \mathcal{H} contains a subgroup isomorphic to the direct product of t copies of $\text{Sym}(k)$ where $tk = n$. Since k is a divisor of n and $k > 1$, the hypothesis on the divisors of n implies $k \geq 5$. Thus $\text{Sym}(k)$ is not a solvable group and \mathcal{H} is not solvable as it contains a nonsolvable subgroup. Thus $f(x)$ is not solvable by radicals.

3. TWO GENERATOR SUBGROUPS OF $\text{Sym}(n)$

Next we apply Theorem 1 to determine the subgroup of $\text{Sym}(n)$ generated by a transposition and one other element. We first consider the case in which